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Aplikace spektrálních metod v syntéze řízení systémů s dopravním zpožděním

Application of spectral methods in control synthesis of time delay systems

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Summary

This document provides an overview of the recent results in the research field of spectrum based analysis and synthesis of time delay systems, which have been achieved with my considerable contribution. Due to the development of modern numerical and optimization methods and with the support of constantly increasing performance of computers and software, this field of infinite-dimensional systems has undergone a substantial development in the last decade. In this text, first, the subject of linear time delay systems is introduced with the focus on the fundamental features of the spectra of poles of neutral and retarded systems. Even though the spectra of these classes of delay systems are both infinite, substantial differences exist in the spectrum distribution and stability. These aspects need to be taken into consideration in both the analysis and control synthesis of the systems. As an example of the retarded time delay system, a model of regenerative chatter in cutting operation is included. Next, results achieved in the field of spectrum analysis and computation are outlined. The main result of this research is an original algorithm QPmR for computation of the extensive spectra of quasi-polynomial characteristic functions of the system. Besides, the extension of the stability determining Michajlov criterion towards neutral systems is presented. The first presented results on the spectrum based synthesis are in the subject of quasi-direct pole placement of retarded systems. The method allows the user to assign the dominant poles of the closed loop system and to guarantee their dominance using optimization methods. It is done by shifting the unassigned rightmost poles from the region of the assigned poles. Consequently, the optimization based synthesis of the spectrum is extended to systems controlled by state derivative (acceleration) feedback. It is shown that arbitrarily small feedback delays can cause instability if the closed loop neutral dynamics is not strongly stable. This stability aspect is considered in the synthesis of the controller. Besides, the application of filter in the feedback loop is applied in order to remove neutrality of the system. Both the analysis and control synthesis are tested on a case study example - model of regenerative chatter.

Souhrn

Je známo, že dopravní (časové) zpoždění má obecně v řídicích systémech destabilizující efekt - čím je časová prodleva akčního zásahu delší, tím obtížněji se systém řídí. Tento jev platí nejenom v technických aplikacích řízení, ale i v mnoha aspektech každodenního života. Efekt zpoždění můžeme například pozorovat ve vývoji počtu studentů jednotlivých univerzitních oborů v závislosti na aktuálních možnostech jejich uplatnění v daných oborech.

Inaugurační spis obsahuje přehled aktuálních výsledků výzkumu v oblasti spektrální analýzy a syntézy řízení systémů s dopravním zpožděním, na kterém jsem se v minulých letech výrazně podílel. Text začíná úvodem do problematiky systémů s dopravním zpožděním, který je zejména zaměřen na spektrální vlastnosti retardovaných a neutrálních systémů. Společným aspektem těchto tříd systémů s dopravním zpožděním, jsou nekonečná spektra jejich vlastních hodnot, reprezentujících jednotlivé módy dynamiky. Systémy ale vykazují významné rozdíly v distribuci spekter pólů a v aspektech týkajících se stability systémů, na které musí být brán zřetel při analýze dynamiky a syntéze řízení. V rámci úvodu je prezentován model s dopravním zpožděním popisující vzájemné působení nože a rotujícího obrobku při soustružení s cílem simulovat potenciálně vznikající netlumené vibrace, tzv. regenerative chatter. Tento model je dále použit k testování navržených spektrálních metod syntézy řízení. První nosné téma spisu se zabývá výsledky, které jsem dosáhl v oblasti analýzy spektrálních vlastností systémů se zpožděním. Hlavním výsledkem je algoritmus QPmR, který byl navržen pro výpočet rozsáhlých spekter kvazi-polynomiálních charakteristických funkcí systémů se zpožděním. Dále je pak zmíněno rozšíření aplikovatelnosti Michajlovova kritéria stability na neutrální systémy. První z prezentovaných metod spektrální syntézy se zabývá syntézou stavového regulátoru pomocí umísťování dominantních pólů nekonečného spektra. Dominance umístěných pólů je zajištěna s využitím optimalizačních metod, pomocí nichž je dosaženo izolování těchto pólů od zbytku spektra. Následně je spektrální optimalizační princip aplikován na syntézu stavového derivačního (akceleračního) regulátoru. V rámci analýzy problému je demonstrován negativní vliv malých dopravních zpoždění na stabilitu uzavřeného regulačního obvodu v případě, že není zajištěna tzv. silná stabilita vzniklého neutrálního systému. Tento aspekt je dále zohledněn v syntéze řízení. Jako alternativní přístup je uvažováno zapojení filtru do zpětné vazby, s jehož pomocí se odstraní neutrální charakter dynamiky systému. Jak analýza vlastností tak i syntéza řízení stavového regulátoru jsou demonstrovány na příkladu modelu interakce nože a obrobku při soustružení.

Keywords

Linear time delay systems, retarded systems, neutral system, difference equation, infinite system spectrum, system stability, strong stability, distribution diagram, state feedback, state derivative feedback, optimization methods

Klíčová slova

Lineární systémy s dopravním zpožděním, retardované systémy, neutrální systémy, diferenční rovnice, nekonečné spektrum systému, stabilita systému, silná stabilita, distribuční diagram, stavový regulátor, stavový derivační regulátor, optimalizační metody

Contents

1 Introduction

This document provides an overview of the results achieved in the spectrum based analysis and synthesis of time delay systems at which I have been involved. The original research results, which were published in a number of prestigious international journals, have been achieved thanks to an extensive scientific collaboration, namely with Prof. Pavel Zítek (CTU in Prague), Prof. Wim Michiels (KU Leuven, Belgium), Prof. Vladimír Kučera (CTU in Prague), Prof. Nejat Olgac (University of Connecticut, USA), Associated Prof. Rifat Sipahi (Northeastern University, Boston USA) and Doc. Didier Henrion (LAAS CNRS Toulouse, France/CTU in Prague). In order to achieve compactness of the text, the results are presented within a broader research context. All the main results of my collaborative research achieved within the presented topics are highlighted with symbol # and provided with a reference to a journal where the results were published. The references are ordered so that the works [1] - [16] are co-authored by me, while the rest are the works related to the state of the art in the particular fields.

1.1 Linear time delay systems

Consider the general description of time delay system in the form

$$\dot{x}(t)+\sum_{k=1}^{N}H_k\dot{x}(t-\tau_k) = A_0x(t)+B_0u(t)+\sum_{k=1}^{N}A_kx(t-\tau_k)+B_ku(t-\tau_k), \quad (1)$$

where $x(t) \in \Re^n$ is the state variable, $u(t) \in \Re^m$ is the vector of inputs, $\tau_k, k = 1, 2 \ldots, N$ are time-delays, and H_k, A_0, A_k, B_0, B_k are real matrices. The system state is determined by function segment $x_t(\tau) = x(t+\tau), \tau \in [-T, 0]$, where T is the largest time delay. The initial conditions are given as $x_0(\tau) = x(\tau), \tau \in [-T, 0]$. Additionally, the function segment of input $u_0(\tau) = u(\tau), \tau \in [-T, 0]$ needs to be defined as a part of the initial conditions too.

In the general form, (1) is a system of neutral type. If $H_k = 0, k = 1, 2 \ldots, N$, the system is of retarded type, which is more commonly used in the engineering applications, e.g. to model heat transfer, chemical and combustion processes, distributed networks, and even systems in economics or biology, see e.g. [23] and references therein. The neutral equations can be used for instance for modeling lossless transmission lines, lossless propagation phenomenon, combustion systems, it arise in boundary controlled hyperbolic PDEs when subjected to small feedback delays and in some implementation schemes of predictive controllers [28].

1.2 System spectrum, stability

Stability of the system (1) can be determined on the basis of the root distribution of the characteristic equation

$$M(s) = \det\left(s\left(I + \sum_{k=1}^{N} H_k e^{-s\tau_k}\right) - A_0 - \sum_{k=1}^{M} A_k e^{-\lambda\tau_k}\right) = 0. \qquad (2)$$

As the equation is transcendental, in general, it has infinitely many roots. The spectra of both retarded and neutral systems are infinite. For the stability evaluation purposes, let us define the spectral abscissa of the system, i.e. the smallest upper bound of the spectrum as

$$\alpha = \max\left\{\Re(s_k), k = 1, 2, .., \infty\right\}. \qquad (3)$$

The system (1) is stable if and only if $\alpha \leq 0$, i.e. all the roots of the equation (2) are located in the left half of the complex plane. As regards the spectrum distribution, considerable differences exist between the retarded and neutral systems. Consider, that all the poles of the system are ordered in a sequence $s_k, k = 1, 2, .., \infty$ according to their magnitude $|s_k|$. The fundamental differences between the retarded and neutral systems can be identified as follows:

- **Retarded systems**: given $a \in \mathbb{R}$, the number of roots satisfying $\Re(s_k) \geq a$ is finite. A direct consequence of this property is that the number of unstable roots is at most finite. Moreover it holds that both $\Re(s_k) \to -\infty$ and $\Im(s_k) \to \infty, (-\infty)$ as $k \to \infty$.

- **Neutral systems**: there exist such $a \in \mathbb{R}, b \in \mathbb{R}, a < b$, both finite, that an infinite number of roots s_k is located within the vertical strip $a < \Re(s_k) < b$. A direct consequence of this property is that the number of unstable roots can be infinite. Besides, the spectral abscissa α can be discontinuous with respect to small changes in the delays. Thus, even infinitesimally small changes in the delays can cause instability.

Taking into considerations these fundamental spectral features, the retarded system can be dealt with in a similar way as the high order delay free systems. The dynamics and stability is determined by the group of dominant, rightmost roots. However, it is not the case of neutral systems, where the instability can be caused by the roots on the very high frequencies. Besides, the discontinuity of α with respect to small delay changes is a risky feature. In order to clarify this stability aspect, let us define the difference equation associated to the system (1) as follows

$$x(t) = \sum_{k=1}^{N} H_k x(t - \tau_k), \qquad (4)$$

and its characteristic equation

$$D(s) = \det(I - \sum_{k=1}^{N} H_k \exp(-s\tau_k)) = 0. \qquad (5)$$

It can be easily proven, see e.g. [6], that the infinite spectrum of (4), i.e. the roots of (5) lie within a certain vertical strip of the complex plane $a < \Re(s_k) < b$ and the spectrum of (1) tend to match the spectrum of (4) in the high frequencies. In order to deal with the problem of sensitivity of the spectral abscissa of difference equations and neutral systems and its stability consequences, the concept of strong stability has been introduced [23, 24]: The difference equation is *strongly stable* if and only if

$$\gamma_0 < 1, \gamma_0 = \max\{r_\sigma(\sum_{k=1}^{N} H_k \exp(\mathrm{i}\theta_k))|\ \theta_k \in [0, 2\pi], 1 \leq k \leq N\}, \qquad (6)$$

where r_σ denotes the spectral radius. If the strong stability condition is satisfied, the neutral system has at most finite number of roots located to the right of the stability boundary. If the condition is not satisfied, the system can be stable for some delay values. However, stability is not strong as even arbitrarily small variations in the delay values destroy the stability. Obviously, evaluation of the strong stability by (6) is not an easy task already for more than two delays. The criterion can either be evaluated numerically via griding the space of parameters $\theta_k, k = 1..N$ and solving the eigenvalue problem for every grid point, as it was done in [6, 8]. Recently Henrion and Vyhlídal [16] proposed an approach based on trigonometric polynomial optimization where the problem is formulated in a form of LMI.

1.3 Introductory example - model of regenerative chatter

The introductory example shows the typical application of time delay systems in mechanical engineering. One of the most important causes of machining instability is the so called regenerative chatter effect, [25, 30, 31]. This regenerative chatter is undesirable due to its adverse effects on surface finish, machining accuracy and also tool life. Because of some external perturbations, the tool starts a damped oscillation relative to the workpiece. After a revolution of the workpiece, the chip thickness will vary at the tool due to this wavy surface. Therefore, the cutting force is dependent on the actual and delayed values of the displacement of the tool. Fig. 1 shows a schematic of the regenerative chatter problem in question. The tool is assumed to be compliant in both the x and y axis, whilst the workpiece is assumed to be rigid.

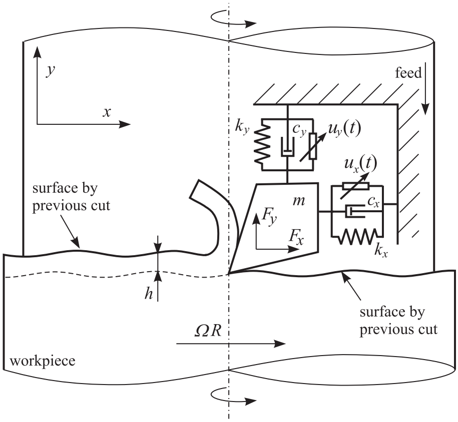

Figure 1: Schematic of a 2-DOF regenerative chatter problem in machining

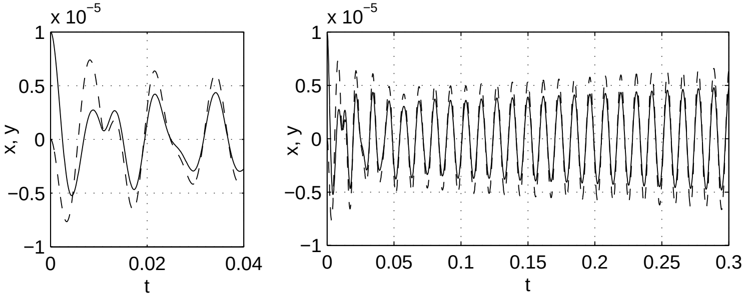

Figure 2: Response of system (7) to initial conditions $z(t) = 0, t \in [-\tau, 0), z(0) = [0, 0, 10^{-5}, 0]^T$, x - dashed, y - solid

It has been shown in [25] that the displacements of the tool can be modeled as a 2-DOF freedom retarded oscillator

$$\dot{z}(t) = A_0 z(t) + A_1 z(t - \tau) + Bu(t) \tag{7}$$

where $z = [x,\ \dot{x},\ y,\ \dot{y}]^T$, $x(t)$, $y(t)$ are displacements of the tool, $u(t) = [u_x(t),\ u_y(t)]^T$, $u_x(t), u_y(t)$ are the control inputs of the system, exerted e.g.

by piezoactuators.,

$$A_0 = \begin{pmatrix} 0 & 1 & 0 & 0 \\ \frac{-k_x}{m} & \frac{-c_x}{m} & \frac{-\kappa_x}{m} & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -\frac{k_y+\kappa_y}{m} & \frac{-c_y}{m} \end{pmatrix}, A_1 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{\kappa_x}{m} & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{\kappa_y}{m} & 0 \end{pmatrix}, \quad (8)$$

$$B = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}^T,$$

As the values of parameters, we consider $m = 5kg$, $c_x = c_y = 1.47$kNs/m, $k_x = k_y = 1.2$MN/m for the modal mass, damping and stiffness respectively, $\kappa_x = 2$MN/m and $\kappa_y = 1.45$MN/m for the cutting force coefficients. The delay $\tau = 0.0105$s is equal to the period of revolution of the workpiece $\tau = \frac{2\pi}{\Omega}$, where Ω is the angular velocity of the workpiece. In Fig. 2, the response of system (7) to initial conditions $z(t) = 0, t \in [-\tau, 0), z(0) = [0, 0, 10^{-5}, 0]^T$ is shown. As can be seen from the increasing amplitudes of the oscillations, the system is unstable. Further on in the text, this conclusion is confirmed by the analysis of the spectrum, see Fig. 5, 7. As can be seen in the figures, the system is unstable with a single couple of unstable poles $\lambda_{1,2} = 1.5 \pm 542.8j$, i.e. $\alpha = 1.5$.

2 Spectrum analysis and computation

There exists several algorithms and software routines for analysis and computation of the spectrum of time delay systems. The most commonly used are *DDE-biftool* [22] and *TRACE-DDE* [20]. In spectrum computation, discretization approaches are used as a rule. An alternative to these approaches is the algorithm *QPmR* proposed by Vyhlídal and Zítek, which is based on mapping and comprehensive analysis of the system characteristic function [14, 1]. The algorithms utilizes the results outlined below.
In the analysis, we consider the quasi-polynomial characteristic function of the system (1) in the following form

$$M(s) = \sum_{j=0}^{N} p_j(s) \mathrm{e}^{-s\alpha_j} \quad (9)$$

where $\alpha_0 > \alpha_1 > ... > \alpha_{N-1} > \alpha_N = 0$ and $p_j(s) = \sum_{k=0}^{m_j} p_{j,k} s^k$ are polynomials in *s* of degree $m_j \leq n$.

2.1 Spectrum distribution

The spectrum asymptotic distribution features of time delay systems have already been studied by Bellman and Cooke (1963), [19]. Determining the principal terms of the system characteristic function $M(s)$, the distribution properties of the roots with high magnitudes has been determined. Let us define

$$g(s) = \sum_{j=1}^{N} p_{j,m_j} s^{m_j} (1 + \varepsilon_{\mathrm{j}}(s)) \mathrm{e}^{s\vartheta_j} \tag{10}$$

which has the same distribution of zeros as (9), where $\vartheta_j = \alpha_0 - \alpha_j$, $0 = \vartheta_0 < \vartheta_1 < ... < \vartheta_{N-1} < \vartheta_N$, $p_{j,m_j} \neq 0$ $(j = 0, 1, ..., N)$ and the functions $\varepsilon_j(s)$ have the property $\lim_{|s|\to\infty} |\varepsilon_j(s)| = 0$. As it has been shown in [19], with the points $P_j = (\vartheta_j, m_j)$, we can define the *Distribution diagram*. It is constructed as upward convex polygonal line *L* over the points $P_j = (\vartheta_j, m_j)$, see Fig. 3 for demonstration. Let the successive segments of *L* be denoted by L_1, L_2,.... L_M, numbered from left to right, and let μ_r denote the slopes of L_r. For each segment of the spectrum distribution diagram L_r with $\mu_r > 0$, a retarded chain with infinitely many roots exists. The segments with $\mu_r = 0$ correspond to the neutral part of the spectrum, which is located in a vertical strip of the complex plane. Based on the *Distribution diagram* and utilizing further results by Bellman and Cooke [19], Vyhlídal and Zítek derived the asymptotic exponentials of the retarded root chains as follows:

Asymptotic exponentials of the root chains (Vyhlídal and Zítek, ***IEEE-TAC***, 2009 [1])

For large magnitudes of $s = \beta + j\omega, \beta \in \mathbb{R}, \omega \in \mathbb{R}^+$, the asymptotic curves of the root chains of (10) can be approximated by the asymptotic exponentials

$$\omega = \exp\left(\frac{c_r - \beta}{\mu_r}\right) \tag{11}$$

where $c_r = \mu_r \ln |w_{r_k}|$, and w_{r_k} is a zero of the polynomial

$$f_r(w) = \sum_{j=0}^{N_r} \bar{p}_j w^{\tilde{m}_j} \tag{12}$$

where $\tilde{m}_j = \bar{m}_j - \bar{m}_0$, $\bar{m}_j$ and $\bar{p}_j$ correspond to those points $P_j = (\vartheta_j, m_j)$ defined for (10) that lie on the particular segment L_r. $N_r + 1$ is the number of points on the segment L_r.

If the system is neutral, $\mu_M = 0$, and the characteristic function of the associated difference equation to the system (1) is given by

$$D(s) = \sum_{j=0}^{N_M} \bar{p}_j \mathrm{e}^{-\mathrm{s}\bar{\alpha}_\mathrm{j}}, \tag{13}$$

where the coefficients $\bar{p}_j$ and delays $\bar{\alpha}_0 > \bar{\alpha}_1 > ... > \bar{\alpha}_{N_M} = 0$ correspond to the points P_j on the segment L_M. Utilizing the results derived in [6], the safe upper bound for the neutral part of the spectra can be computed in the following way:

Safe upper bound of the spectrum of the difference equation (Michiels and Vyhlídal, ***Automatica***, 2005, [6])
The safe upper bound C_D of the spectrum of the associated difference equation is determined as a single zero of the strictly decreasing function

$$c \in \mathbb{R} \rightarrow \sum_{j=0}^{N_{M-1}} \left| \frac{\bar{p}_j}{\bar{p}_{N_M}} \right| \mathrm{e}^{-\mathrm{c}\bar{\alpha}_\mathrm{j}} - 1. \tag{14}$$

2.2 QPmR: Quasi-polynomial mapping based rootfinder

The QPmR algorithm has been designed by Vyhlídal and Zítek in [14] and extended in [1] to compute all zeros of a quasi-polynomial in a given region $D = [\beta_{\min}, \beta_{\max}] \times j[\omega_{\min}, \omega_{\max}]$ of the complex plane. The main idea of the algorithm is given as follows:

Quasi-polynomial root mapping (Vyhlídal and Zítek, ***IEEE-TAC***, 2009, [1])
Consider $s = \beta + j\omega, \beta \in \mathbb{R}, \omega \in \mathbb{R}^+$, the characteristic quasi-polynomial $M(s)$ can be split into $R(\beta, \omega) = \Re(M(\beta + j\omega))$ and $I(\beta, \omega) = \Im(M(\beta + j\omega))$. Consequently, the characteristic equation $M(s) = 0$ can be split into

$$\begin{aligned} R(\beta, \omega) = 0, \\ I(\beta, \omega) = 0. \end{aligned} \tag{15}$$

Analytic solution of the set (15) is possible only for the most simple quasi-polynomials. Application of standard numerical equation solvers is possible, but it is often limited by the complexity of the problem. In QPmR algorithm, the zero-level curve tracing algorithm is applied in order to approximate the contours in the plane $\beta \times \omega$ described by

(15). Consequently, the intersection points of the contours are determined providing the first approximation of the root positions. Finally, the accuracy of the root is increased by applying the *Newton's method*.

The Quasipolynomial rootfinder *QPmR* described above has been implemented as a Matlab function. Next to determining the position of the roots, it also performs the additional analysis of the spectrum: determining the asymptotic exponentials of the root chains, computing spectrum of the associated difference equation and its safe upper bound, if the system is neutral. The rootfinder has become one of the tools for the spectrum analysis (altogether, there are more than 15 references to the papers [14] and [1] on the ISI Web of Science). For example, in [9] (Olgac, Vyhlídal and Sipahi, *SICON*, 2008), the method was applied to determine the stability maps in the delay domain. The following example demonstrates the application of the algorithm.

2.2.1 Example - application of QPmR algorithm

Consider the following quasi-polynomial

$$M(s) = (s^4 + 2s^2) + (0.5s^4 - 3)\mathrm{e}^{-s} + (0.6s^4 + 0.8s^2)\mathrm{e}^{-2s} + 0.6s^2\mathrm{e}^{-3s} + (-3s^3 + s^2)\mathrm{e}^{-4s} + (3s^2 - 4)\mathrm{e}^{-6s} + \mathrm{e}^{-8s}. \tag{16}$$

The task is to compute all the roots located in the region given by $\beta \in [-6, 4]$ and $\omega \in [0, 150]$. The results are shown in Fig. 3, which consists of quasi-polynomial zeros, asymptotic exponentials of the root chains, safe upper bound and zeros of the associated difference equation.

2.3 Michajlov criterion for neutral systems

It is well known that for stability assessment of delay free and retarded systems, Michajlov criterion can be used [33]. The method, which is based on direct application of the argument increment principle, checks whether the right half of the complex plane is free of function zeros.
Consider the characteristic function $M(s)$ of the system is of polynomial or retarded quasi-polynomial form. Then the system is stable if and only if the Michajlov criterion

$$\lim_{r\to\infty} \Delta \arg M(\jmath\omega)|_{\omega\in[0,r)} = n\frac{\pi}{2}. \tag{17}$$

is satisfied. However, as has been shown by Vyhlídal and Zítek in [2], the criterion cannot be directly applied to neutral quasi-polynomials as the limit at infinity does not exist. Consequently, the modification of the criterion for evaluating stability of the neutral systems has been proposed as follows:

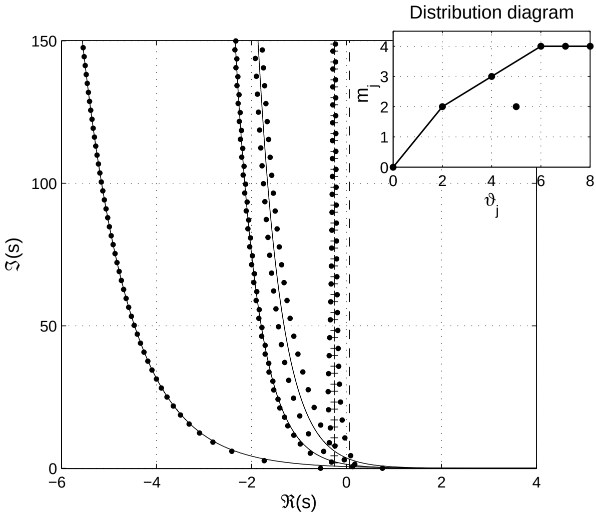

Figure 3: Result of the QPmR Matlab function applied to the quasipolynomial (16). Black dots - quasi-polynomial zeros, solid lines - asymptotic exponentials of the root chains, dashed line - safe upper bound, crosses - zeros of the associated difference equation

- **Modification of Michajlov criterion for neutral systems** (Vyhlídal and Zítek, ***IEEE TAC***, 2009b, [2])
  Consider a system (1) with characteristic quasi-polynomial (9). If $M(\jmath\omega) \neq 0$ for any $\omega \in \mathbb{R}^+$, the function $M(s)$ has no zeros in the right half of the s-plane if and only if the argument of $M(\jmath\omega)$ (beginning at zero for $\omega = 0$) reaches the increment for $\omega \to \infty$ lying within the band given by the condition

$$n\frac{\pi}{2} - \Phi \leq \Delta \arg M(\jmath\omega)|_{\omega\in[0,\infty)} \leq n\frac{\pi}{2} + \Phi \tag{18}$$

where

$$\Phi = \arcsin\left(\sum_{i=0}^{N_{M-1}} \left|\frac{\bar{p}_j}{\bar{p}_{N_M}}\right|\right). \tag{19}$$

The symbols in (19) have the same meaning as in (13). Obviously, the necessary condition for determining the angle Φ is that $\sum_{i=0}^{N_{M-1}} \left|\frac{\bar{p}_j}{\bar{p}_{N_M}}\right| \leq 1$. In fact, this is the strong stability condition for scalar systems [23, 6].

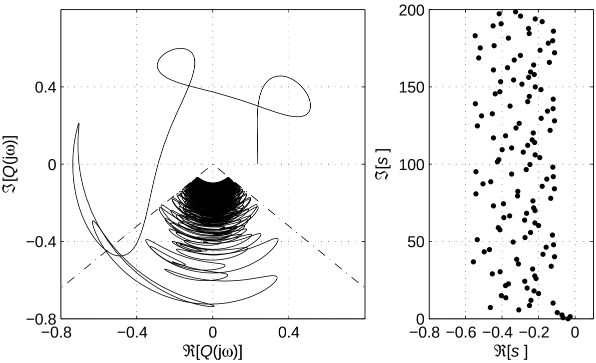

Figure 4: The transformation (21) of (20), $\omega \in [0, 2000]$, dash-dotted line - stability sector determined by the vertex angle Φ; right - a part of the rightmost zeros of (20)

If the condition (18) is satisfied, system (1) is stable and the stability is strong (insensitive to small changes in the delays).

2.3.1 Example - Michajlov criterion application

Let us investigate the stability of the system with the characteristic quasi-polynomial

$$\begin{gathered} M(s) = s^3(1 + 0.23\mathrm{e}^{-s} - 0.25\mathrm{e}^{-\pi s} + 0.3\mathrm{e}^{-4s}) + \\ + s^2(1.3 - 0.3\mathrm{e}^{-5s}) + 2.2s + 1 - 0.7\mathrm{e}^{-7s}. \end{gathered} \tag{20}$$

By applying the mapping

$$Q(s) = \frac{M(s)}{1 + |M(s)|^{1.1}}, \tag{21}$$

it can be seen from the hodograph in Fig. 4 that, obviously

$$3\frac{\pi}{2} - \Phi \leq \Delta \arg Q(\jmath\omega)|_{\omega \in [0,\infty)} \leq 3\frac{\pi}{2} + \Phi \tag{22}$$

where $\Phi = \arcsin(0.23 + 0.25 + 0.3) = 51.3^{\circ}$. Thus the condition (18) is satisfied. As also $M(\jmath\omega) \neq 0$ for the whole frequency range, the system is stable with all roots in the left half of the complex plane, see the right part of Fig. 4.

3 Quasi-direct pole placement

The quasi-direct pole placement method proposed in [5] is inspired by the classical pole placement method for systems without delays. For $n-th$ order SISO systems, the pole placement method allows the assignment of n poles to desired positions and, accordingly, the gain values of a state feedback controller are computed, see e.g. [26]. As it has been shown in [10, 11, 15, 27, 32, 33], the same idea can be applied to adjust the dynamics of time delay systems. However, such a direct assignment of poles has considerable limitations induced mainly by the infinite system's spectrum and the limited degrees of freedom in the controller parameter space.
We consider a general retarded system of the form

$$\dot{x}(t) = \int_0^T dA(\tau)\, x(t-\tau) + \int_0^T dB(\tau)\, u(t-\tau), \tag{23}$$

where $x \in \mathbb{R}^n$ is the vector of state variables, $u \in \mathbb{R}$ is the system's input, τ is the delay variable, which is constrained by the relation $0 \leq \tau \leq T$. The functional matrices $\tau \mapsto A(\tau) \in \mathbb{R}^{n\times n}$, $\tau \mapsto B(\tau) \in \mathbb{R}^{n\times 1}$ describe the distribution of the delay and cover both multiple lumped (pointwise) and distributed delays, [33]. Consider a feedback controller of the form

$$u(t) = -Kx(t), \tag{24}$$

where $K := [k_1\ k_2\ \cdots\ k_p]$ contains the controller parameters to be determined. The stability properties of the feedback system (23) and (24) are determined by the roots of the characteristic equation

$$M(s) = \det\left(sI - A(s) - \sum_{j=1}^{p} k_j B(s)\right) = 0, \tag{25}$$

where $A(s) = \int_0^T \exp(s\tau) dA(\tau)$, $B(s) = \int_0^T \exp(s\tau) dB(\tau)$. Under the condition $\det(sI - A(\lambda)) \neq 0$, the characteristic equation can be written as

$$\begin{aligned} &\det\left(I - \textstyle\sum_{j=1}^{p} k_j B(s)(sI - A(s))^{-1}\right) = 0 \\ &\Leftrightarrow 1 - \textstyle\sum_{j=1}^{p} (sI - A(s))^{-1} B(s) k_j = 0. \end{aligned}$$

Assigning a real pole to the location c yields the following constraint on the gain values:

$$\sum_{j=1}^{p} (cI - A(c))^{-1} B(c)\, k_j = 1.$$

In this way, assigning $0 \le m \le p$ poles to $\lambda_1, \ldots, \lambda_m$ eventually results in a set of m constraints which can be written in the form

$$SK = R, \tag{26}$$

where $S \in \mathbb{R}^{m \times p}$ and $R \in \mathbb{R}^{m \times 1}$. Using the singular value decomposition $S = U \Sigma V^*$, where $(\cdot)^*$ denotes the complex conjugate transpose, condition (26) can be transformed into the following form

$$\Sigma L = U^* R \tag{27}$$

where $L = V^* K$. Letting $L = [l_1 \ \cdots \ l_p]^T$, $U^* R = [\bar{r}_1 \ \cdots \ \bar{r}_m]^T$, $\Sigma = [\mathrm{diag}(\sigma_1, \ldots, \sigma_m) \ \mathbf{0}]$, and assuming that S is of full (row) rank, we finally get the following expression for the parameters $l_j, \ j = 1, \ldots, m$, corresponding to the assigned poles $\lambda_i, i = 1..m$:

$$\left\{ \begin{array}{ccc} l_1 & = & \bar{r}_1/\sigma_1, \\ l_2 & = & \bar{r}_2/\sigma_2, \\ \vdots & & \vdots \\ l_m & = & \bar{r}_m/\sigma_m. \end{array} \right. \tag{28}$$

- \# **Controller parameterization** (Michiels, Vyhlídal and Zítek, ***J Process Contr.***, 2010, [5])
  Assume that S has full row rank. Considering parameters $l_1, \ldots, l_m$ as fixed according to (28), and $l_{m+1}, \ldots, l_p$ being kept as free parameters, one obtains a parametrization of all controllers that assign m poles to $\lambda_1, \ldots, \lambda_m$.

The next step of the pole placement procedure is as follows. Assume that m poles are assigned as described in (28). Thus, the parameters $l_1, \ldots, l_m$ are fixed and the parameters $l_{m+1}, \ldots, l_p$ are available for further adjustment of the system's spectrum. Applying the algorithm described in [32], the parameters $l_{m+1}, \ldots, l_p$ are to be used to push the other rightmost poles as far as possible to the left. However, unlike in [32], where the spectral abscissa α is minimized, we minimize instead the function

$$\left. \begin{array}{l} \bar{\alpha}(l_{m+1}, \ldots, l_p) = \sup \{ \Re(\lambda) : \\ \frac{\det\left(\lambda I - A(\lambda) - \sum_{j=1}^{p} k_j(l_{m+1}, \ldots, l_p) B(\lambda)\right)}{\Pi_{j=1}^{m}(\lambda - \lambda_j)} = 0 \end{array} \right\}. \tag{29}$$

Technically, the evaluation of $\bar{\alpha}$ is rather straightforward. First, the rightmost characteristic roots of (23) and (24) are computed. In the next step, the *invariant* characteristic roots $\lambda_1, \ldots, \lambda_m$ are removed from the spectrum and the objective function is obtained as $\bar{\alpha} = \sup_{i>m}(\Re(\lambda_i))$. In order to perform the minimization task for the objective function (29), which is, in general, non-smooth and non-convex, hybrid algorithm HANSO [29] combining BFGS and gradient sampling optimization method can be used. The overall algorithm can then be summarized as follows:

Algorithm - Quasi-direct pole placement (Michiels, Vyhlídal and Zítek, ***J Process Contr.***, 2010, [5])
Consider feedback system (23) and (24).

1. Select poles $\lambda_1, \ldots, \lambda_m$, $m < n$ to be assigned.
2. Compute the parameters $l_1, \ldots, l_m$ as described in (28).
3. Minimize the function $(l_{m+1}, \ldots, l_p) \mapsto \bar{\alpha}(l_{m+1}, \ldots, l_p)$.
4. If $\min \bar{\alpha} < \min_{1 \leq j \leq m} \Re(\lambda_j)$, accept the result and transform L to K by $K = VL$. In the other case (that is, the assigned poles cannot be separated from the remainder of the spectrum), select different assigned poles and go to step (1).

3.1 Example - quasi-direct pole placement

Consider the model of the regenerative chatter given by (7). The task is to perform pole placement for the system using the state feedback

$$u_y(t) = -Kz(t). \tag{30}$$

Assigning the couple of poles $\lambda_{1,2} = -100 \pm j600$ as described in the Quasi-direct pole placement algorithm, we obtain gain parameters

$$K = \left[1.9301 \cdot 10^5, 1.3249 \cdot 10^2, 1.4555 \cdot 10^6, 1.0631 \cdot 10^3\right],$$

resulting in favorable improvement of the system stability, as demonstrated in Figures 5, 6. As can be seen from the spectrum, the system has been safely stabilized and $\bar{\alpha} = -131.5 < \Re(\lambda_{1,2})$. Thus, the assigned poles are truly the dominant poles. The considerable improvement can also be seen in the response to initial conditions $z(t), t \in [-\tau, 0), z(0) = [0, 0, 10^{-5}, 0]^T$ in Fig. 6, when compared with unstable system response in Fig. 2. Let us remark that the $HANSO$ algorithm [29] was applied to perform the optimization task.

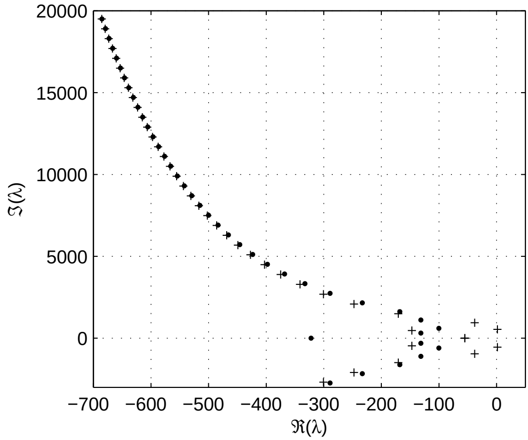

Figure 5: Rightmost spectrum of the regenerative chatter model (7), crosses - open loop system spectrum, dots - closed loop system spectrum controlled by (30), assigned poles: $\lambda_{1,2} = -100 \pm j600$.

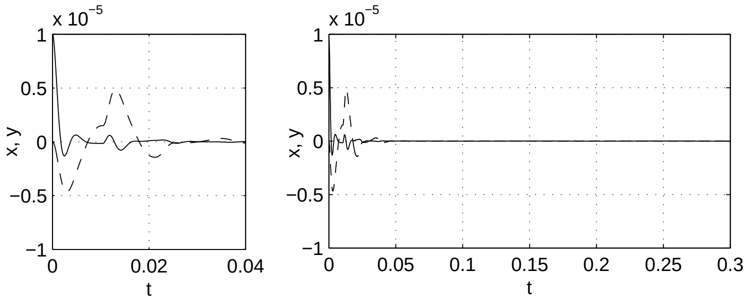

Figure 6: Response of the closed loop system (7)-(30) to initial conditions $z(t) = 0, t \in [-\tau, 0), z(0) = [0, 0, 10^{-5}, 0]^T$, x - dashed, y - solid

4 State derivative feedback

This section presents results achieved in the spectrum based analysis and synthesis of state derivative feedback with small feedback delays. The motivation for state derivative feedback comes from controlled vibration suppression of mechanical systems [17, 18]. In vibration control problems, accelerometers are typically used for measuring the system motion. As a result accelerations and velocities are the sensed variables as opposed to displacements.

4.1 Effect of small delays in the feedback loop

It is well known that small delays systematically originate from the latency effect of implementing the measurements and control actions, occurring, e.g., as a consequence of computational delays, delays arising from AD-DA conversion or communication delays. As a rule, such delays are very small compared to the dominant modes of the system, which justifies to neglect them in the feedback design in most applications. However, it has been shown in our papers [4, 8] that such delays cannot always be safely neglected if state derivatives are used for feedback, as even arbitrarily small implementation delays may cause instability. We consider the delay free system in the form

$$\dot{x}(t) = Ax(t) + Bu(t), \tag{31}$$

where $x \in \mathbb{R}^n$ is the vector of state variables, $u \in \mathbb{R}^{n_u}$ is the vector of inputs and A, B are constant coefficient matrices. The proportional-derivative (PD) state controller is considered as follows

$$u(t) = -K_p\, x(t) - K_d\, \dot{x}(t) \tag{32}$$

where $K_p \in \mathbb{R}^{n_u \times n}$ and $K_d \in \mathbb{R}^{n_u \times n}$ are the feedback gain matrices. The closed loop system (31)-(32) is given by

$$\dot{x}(t) = (I + BK_d)^{-1}(A - BK_p)x(t) \tag{33}$$

Obviously, the n eigenvalues of the matrix $(I+BK_d)^{-1}(A-BK_p)$ determine the stability and the dynamic behavior. As it has been shown by Abdelazis and Valášek [17, 18], the necessary condition for stabilizability of the system (31) is that $\det(A) \neq 0$, i.e. the system does not have a pole at the origin of the complex plane.

Consider that delays appear in the feedback paths of the system (31)-(32) as follows: a delay τ_{u_k} on the k-th component of input u, $1 \leq k \leq n_u$, a delay $\tau_{\dot{x}_l}$ in the measurement of the l-th component of $\dot{x}$, and a delay τ_{x_l} in the measurement of the l-th component of $x,\ 1 \leq l \leq n$. The closed-loop system becomes

$$\dot{x}(t) + \sum_{k=1}^{n_u} BE_k \sum_{l=1}^{n} K_d F_l \dot{x}(t - \tau_{u_k} - \tau_{\dot{x}_l}) = \\ Ax(t) - \sum_{k=1}^{n_u} BE_k \sum_{l=1}^{n} K_p F_l x(t - \tau_{u_k} - \tau_{x_l}), \tag{34}$$

where $E_k = [e^k_{i,j}] \in \mathbb{R}^{n_u \times n_u}$ and $F_l = [f^l_{i,j}] \in \mathbb{R}^{n \times n}$ satisfy

$$e^k_{i,j} = \begin{cases} 1, & i = j = k \\ 0, & \text{otherwise} \end{cases}, \quad f^l_{i,j} = \begin{cases} 1, & i = j = l \\ 0, & \text{otherwise} \end{cases}, \tag{35}$$

for $k = 1, \ldots, n_u$ and $l = 1, \ldots, n$. Notice that if $K_d \neq 0$, the system (34) is a time-delay system of neutral type, which induces complications w.r.t. stability issues. As it has been discussed above, the strong stability needs to be guaranteed for neutral systems. Even if the closed loop system (33) is stable, the stability can be lost due to small delays in the state derivative feedback paths. However, as it has been shown in [7], if dependencies in the delays occur, the strong stability condition needs to be modified as follows:

Strong stability condition on the gain parameters (Michiels W., T. Vyhlídal, P. Zítek, H. Nijmeijer and D. Henrion, ***SICON***, 2009, [7])
Assume the system (31) is stabilized with the control law (32). If the feedback gain K_d is such that

$$\gamma_0(K_d) := \max\left\{\alpha\left(-\sum_{k=1}^{n_u} BE_k \sum_{l=1}^{n} K_d F_l e^{i(\mu_k+\nu_l)}\right) : \vec{\mu} \in [0,\ 2\pi]^{n_u},\ \vec{\nu} \in [0,\ 2\pi]^{n}\right\} < 1, \tag{36}$$

then the exponential stability of the closed-loop system is robust against small feedback delays.

4.2 Filtered derivative feedback

From the practical point of view, the neutrality induced by the state derivative feedback should be considered as the worst (limit) case when we have both the ideally true model and the ideal controller. In practice, any filtering effect in the feedback loop is likely to remove the neutrality. This is demonstrated on using the application of filtered derivative feedback:

Filtered state derivative feedback (Vyhlídal T., W. Michiels, P. Zítek and P. McGahan, ***Contr. Eng. Practice***, 2009, [4])
When applying a first order filter to (32), the controller becomes:

$$T\dot{u}(t) + u(t) = -K_p\, x(t) - K_d\, \dot{x}(t) \tag{37}$$

where $T = 1/\omega_f$ is the time constant of the filter, and ω_f is its cutoff frequency.

The closed loop system under the influence of the feedback delays then changes from (34) to

$$\dot{z}(t) + \sum_{k=1}^{n} \begin{bmatrix} 0 & 0 \\ \frac{1}{T}K_d F_k & 0 \end{bmatrix} \dot{z}(t - \tau_{\dot{x}_k}) = \begin{bmatrix} A & 0 \\ 0 & -\frac{1}{T}I \end{bmatrix} z(t) +$$
$$+ \sum_{k=1}^{n_u} \begin{bmatrix} 0 & BE_k \\ 0 & 0 \end{bmatrix} z(t - \tau_{u_k}) \sum_{k=1}^{n} \begin{bmatrix} 0 & 0 \\ -\frac{1}{T}K_p F_k & 0 \end{bmatrix} z(t - \tau_{x_k}), \tag{38}$$

with $E_k,\ 1 \le k \le n_u$ and $F_k,\ 1 \le k \le n$ defined in (35). This system corresponds to the degenerate case where the characteristic function of the associated difference equation $D(s) = 1$. If K_d and T are such that the delay free system is asymptotically stable, the stability is *preserved* for small values of the delays.

Although stability is always preserved for sufficiently small delays, the maximal allowable delays tend to zero as $T \to 0$ whenever $\gamma_0(K_d) > 1$. This implies an inherent trade-off in determining the cut-off frequency of the filter: in order not to affect the nominal, delay free behavior too much the cut-off frequency should be sufficiently large. However, if the cut-off frequency is too large, then the delay margin may be unacceptably small.

4.3 State derivative feedback design for retarded systems

As a possible way of the state derivative feedback design, optimization based spectral synthesis can be applied, similarly as in the section on quasi-direct pole placement. As the nominal system, let us consider the retarded system of the form

$$\dot{x}(t) = A_0 x(t) + \sum_{k=1}^{M} A_k x(t - \tau_k) + Bu(t) \tag{39}$$

where $x(t) \in \Re^n$ $A_k \in \Re^{n\times n},\ k = 0, 1, \ldots, M$, $B \in \Re^{n\times n_u}$, and $\tau_k > 0,\ k = 1, \ldots, M$, are time delays. We consider a state derivative feedback of the form

$$u(t) = -K\dot{x}(t). \tag{40}$$

Consider that delays appear in the feedback paths of the system (39)-(40). If we assume that there is a delay τ_{u_k} on the k-th component of input u, $1 \le k \le n_u$ and a delay $\tau_{\dot{x}_l}$ in the measurement of the l-th component of $\dot{x}$

then the closed-loop system becomes:

$$\dot{x}(t)+\sum_{k=1}^{n_u} BE_k \sum_{l=1}^{n} KF_l \dot{x}(t-\tau_{u_k}-\tau_{\dot{x}_l})=\sum_{k=1}^{M} A_k x(t-\tau_k) \qquad (41)$$

where $E_k = [e^k_{i,j}] \in \mathbb{R}^{n_u \times n_u}$ and $F_l = [f^l_{i,j}] \in \mathbb{R}^{n\times n}$ are defined as in (35). Analogously as in (34), the system (41) is of neutral type. Thus, even if the system is stabilized by the feedback control law (40), the strong stability condition (36) needs to be satisfied in order to preserve stability for small delay changes in the feedback loop.

4.3.1 The optimization problem

Let us define the spectral abscissa as the real part of the rightmost root of the system (39)

$$\alpha(K) := \sup\left\{\Re(s_k), k = 1, 2, .., \infty\right\}.$$

The objective is to stabilize the system via minimizing the spectral abscissa $\alpha(K)$. However, the constraint $\gamma_0(K) < 1$ needs to be satisfied as well. A solution of this synthesis problem can be found by solving the *constrained* optimization problem

$$\min_K \alpha\left(K\right), \text{subject to}\ \gamma_0(K) < \gamma, \qquad (42)$$

where $\gamma < 1$. In practical applications, due to robustness reasons, we should avoid settings of K for which $\gamma_0(K)$ is close to 1 even. On the other hand, too small choice of γ value would result in too conservative solution. From our experience it is advisable to set $\gamma \in [0.6, 0.9]$. Since the available methods for eigenvalue optimization problems [21, 32] can only deal with unconstrained problems, a natural way to handle the constraint in (42) consists of using a *barrier method*, as outlined below.

\# **Neutral spectrum optimization using barrier method** (Vyhlídal, Michiels, McGahan, ***IMA J. Math. Contr. and Opt.***, 2010, [3])
The spectrum optimization problem with the aim to strongly stabilize the system (39) by the state derivative feedback controller (40) can be formulated as follows:

1. Find a a feasible point, i.e. gain values satisfying the constraint. If the feasible set is nonempty such a point can be found by solving

$$\min_K \gamma_0(K). \qquad (43)$$

2. Solve in the next step the *unconstrained* optimization problem

$$\min_K \{f(K)\}, f(K) = \alpha(K) - r \log(\gamma - \gamma_0(K)) \qquad (44)$$

where $r > 0$ is a small number.

For the optimization task of the objective function, which is in general non-smooth and non-convex, the combined BFGS - gradient sampling algorithm based on the code HANSO can be used, see [29]. Let us remark that the optimization routines work with the gradients $\vec{\nabla}\gamma_0(K)$ and

$$\vec{\nabla}\alpha(K) + \frac{r}{\gamma - \gamma_0(K)} \vec{\nabla}\gamma_0(K), \qquad (45)$$

that needs to be determined, e.g. numerically using method of finite differences.

4.4 Example - state derivative feedback design

Consider the model of the regenerative chatter given by (7) with a couple of dominant unstable poles and a controller of the form

$$u(t) = -K\dot{z}(t), \qquad (46)$$

where $K \in \mathbb{R}^{2\times 4}$ is the feedback gain. Let us recall that the signals in $\dot{z}$ are considered as measured by acceleration and velocity sensors. First, let us minimize the spectral abscissa $\alpha(K)$ in the space of parameters K with not paying attention to the strong stability criterion (36). Using the optimization tool HANSO, we achieve results given in the first row of the Table 1. As can be seen, the system has been safely stabilized. However, the system is not strongly stable as $\gamma_0(K) = 1.850$. Secondly, let us minimize the objective function (44), considering $\gamma_0(K_D) < 0.8$ and the *barrier parameter* $r = 0.1$. For the evaluation of the gradient of the objective function (45), notice, that $\nabla\alpha(K)$ is computed analytically and the gradient $\nabla\gamma_0(K_D)$ is computed numerically by the method of finite differences. For the minimization task, again, the HANSO algorithm is used. The results are given in the second row of Table 1. In this case, the nominal system is both asymptotically stable and the stability is strong with respect to small feedback delays.
For the analysis of the final settings given in Table 1, we provide a comparison of the spectrum distributions in Fig. 7. Next to the nominal closed loop system (without feedback delays) we consider closed loop system in the form (41) with the following small delays in the feedback loops

$$\begin{gathered}\tau_{u_1} = e^2\, 10^{-5}, \tau_{u_2} = 2\pi\, 10^{-5}, \tau_x = 10^{-5}, \\ \tau_y = 3\, 10^{-5}, \tau_{\dot{x}} = \sqrt{2}\, 10^{-5}, \tau_{\dot{y}} = 2\, 10^{-5}\end{gathered} \qquad (47)$$

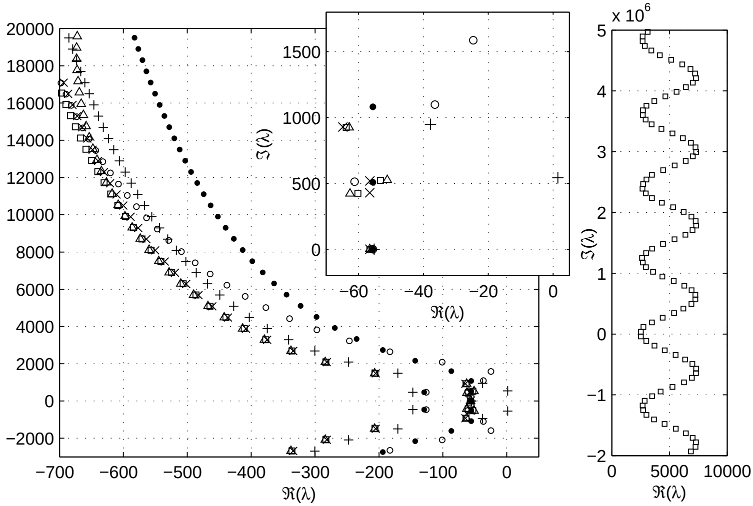

Figure 7: Rightmost spectra of the regenerative chatter model. Spectrum of the nominal system (7) [crosses]; spectrum of the closed loop system (7)-(46) (with no feedback delays) with K_α [black circles] anf with K_f [x]; spectrum of the closed loop system (7)-(46) and with small feedback delays (47) with K_α [circles] and with K_f [squares]; spectrum of the closed loop system (7)-(48) (filterred derivative feedback) with K_f and $T_f = 0.00005$ [triangles]

Minima	γ_0	K
$\alpha(K) = -56.5$	1.850	$K_\alpha = \begin{bmatrix} 234.24 & -0.507 & -795.64 & 0.899 \\ 200.05 & -0.51229 & -344.76 & 1.506 \end{bmatrix}$
$f(K) = -55.18$	0.784	$K_f = \begin{bmatrix} 11.3 & -0.728 & 9.28 & -0.135 \\ -13.3 & -0.623 & -31.26 & 0.728 \end{bmatrix}$

Table 1: Results of the state derivative feedback design by eigenvalue optimization via HANSO. First row - minimization of spectral abscissa α; second row - minimization of the function f in (44)

As it has already been mentioned, the uncontrolled nominal system is unstable due to a couple of poles located to the right of the stability boundary. As can be seen, both applied optimization approaches achieved very similar distribution of the spectra. In both cases, the real parts of few rightmost poles are located close to the spectral abscissa, whereas the rest of the infinite spectrum follows the asymptotic root chain of the retarded system. If small

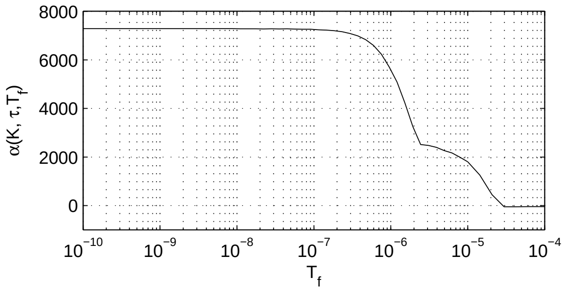

Figure 8: Spectral abscissa of the closed loop system (7)-(48) (filterred derivative feedback with setting K_α) considering feedback delays (47) with respect to the value of the time constant of the filter T_f

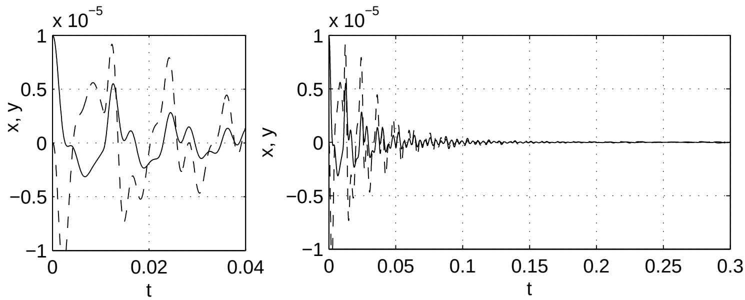

Figure 9: Response of the closed loop system (7)-(46) considering feedback delays (47) to the initial conditions $z(t) = 0, t \in [-\tau, 0), z(0) = [0, 0, 10^{-5}, 0]^T$

feedback delays are introduced, the distribution of poles in the (so far) dominant region change only slightly in both cases. However, in fact, the stability is preserved only for the case with setting K_f. The other case with K_α becomes unstable, with infinitely many unstable roots and the spectral abscissa changes from $\alpha = -56.5$ to $\alpha = 7288$, see the new rightmost spectrum in Fig. 7 - right. As it has been described in Section 4.2, the neutrality of the system can be removed by applying the filtered derivative feedback

$$T_f \dot{u}(t) + u(t) = K\dot{z}(t). \tag{48}$$

However, the time constant of the filter needs to be large enough to move the spectral abscissa behind the stability boundary. In Fig. 8, we show the dependence of the spectral abscissa of the closed loop system with K_α with

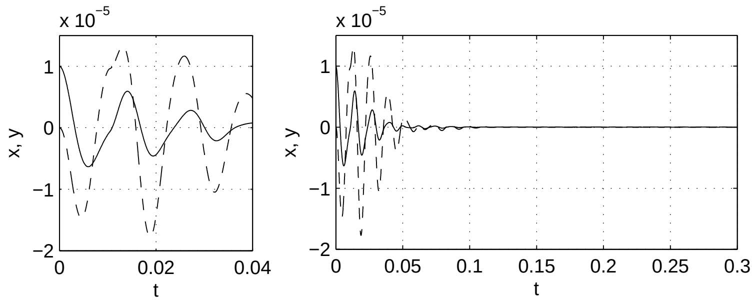

Figure 10: Response of the closed loop system (7)-(48) considering feedback delays (47) to the initial conditions $z(t) = 0, t \in [-\tau, 0), z(0) = [0, 0, 10^{-5}, 0]^T$, $T_f = 0.00005$

respect to the time constant of the filter T_f. As can be seen, the closed loop system is stabilized for $T_f > 0.00003$. In Fig. 7 the spectrum of closed loop system (7)-(46) with setting K_α, $T_f = 0.00005$ and with feedback delays (47) is shown. As can be seen, the dominant poles stayed fairly close to the spectrum of the nominal case. The results of the spectrum based analysis are confirmed by the system responses to the initial conditions $z(t) = 0, t \in [-\tau, 0), z(0) = [0, 0, 10^{-5}, 0]^T$ under the influence of feedback delays (47). As can be seen in Fig. 9, the response with strongly stable feedback K_f, is stable. It also applies if the strongly unstable setting K_α with stabilizable first order filter with $T_f = 0.00005$ is applied, see Fig. 10. However, as can be seen in Fig. 11, for $T_f = 0.000024$, the response is unstable due to emerging high-frequency oscillations.

4.5 Remark on p-stability

As it results form both the spectrum and response based analysis in the above example, the application of the filter, which turns the dynamics from neutral to retarded-like one, can stabilize the closed loop system with strongly unstable derivative feedback. However, the value of the time constant of the filter needs to be designed properly as not every value stabilizes the system. Obviously, for the given case study of the model of regenerative chatter, most likely, already the filter embedded in the smart acceleration sensors, which is not modeled in fact, would remove the dangerous high-frequency oscillations. On the other hand, it needs to be emphasized that this conclusion cannot be generalized. As has been demonstrated in (Michiels, Vyhlídal, et.al, ***SI-***

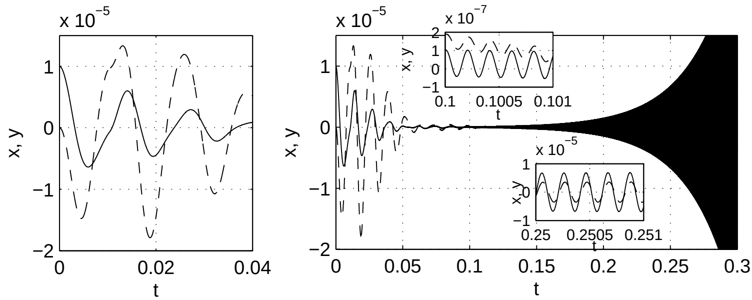

Figure 11: Response of the closed loop system (7)-(48) considering feedback delays (47) to the initial conditions $z(t) = 0, t \in [-\tau, 0), z(0) = [0, 0, 10^{-5}, 0]^T$, $T_f = 0.000024$

CON, 2009, [8]), in some cases, unmodeled dynamics (neglected actuator or sensor dynamics) can have destabilizing effect even if the delays are not considered in the feedback loop. In the mentioned paper, the notion of p-stability is introduced under which the closed loop system (31)-(40) is robust against small modeling and implementation errors. For example, it is shown that the system (31) with an odd number of unstable poles, i.e. $\det(-A) < 0$ cannot be safely stabilized by state derivative feedback even if it is equipped with a low-pass filter. Besides, the necessary condition for p-stabilizability by the filtered state derivative feedback with small values of T_f is that the matrix $(-I - BK_d)$ has all eigenvalues in the left half of the complex plane.

5 Conclusions

The main results I have achieved in the subject of spectrum based analysis and synthesis can be summarized as follows. Since the beginning of my research carrier, I have focused on developing algorithms for spectrum analysis and computation. Our algorithm QPmR for computation of the spectrum of quasi-polynomials has received considerable attention in the community of time delay systems and has become one of the standard tools for spectrum computation. I have also achieved interesting results in the frequency based analysis, for example in the extension of Michajlov stability criterion for application to neutral systems. In collaboration with Prof. Pavel Zítek and Prof. Wim Michiels, we have achieved original results in the spectrum based synthesis of state and state-derivative feedback controllers. The presented optimization based quasi-direct pole placement method for SISO retarded systems is a direct extension of classical pole placement algorithm towards the systems with infinite spectrum. In this method, the infinite dimensional system is controlled by static state feedback as usual, which makes the controller implementation much easier compared to implementing the functional feedback. The same applies also for presented state derivative feedback design, which is motivated by using accelerometers to measure the system motion, particularly in vibration suppression control systems. Surprisingly, as it results from the theoretical analysis, even infinitesimally small feedback delays, e.g. of communication origin, may play important role in the closed loop system stability. Therefore, the concept of strong stability needs to be taken into consideration in the feedback design. Next it is demonstrated that, in some cases, the neutrality induced by small feedback delays can be removed by a first order filter. The results are demonstrated on the model of regenerative chatter in machining.

5.1 Further research

As regards my actual and further research in the field, the main directions are the following. Currently, the assessment of strong stability criterion using polynomial optimization approaches is being solved. The work will also continue in studying the synthesis and control design of state feedback, both proportional and derivative. Particular attention will be paid to the practical evaluation of the derived results. Recently, we have also started to work in the subject of delay based signal shapers for compensating undesirable oscillatory modes of flexible structures with the aim to involve spectral methods in their design.

5.2 Projects

Next to the theory of time delay systems done in the framework of the project **Centre for Applied Cybernetics**, I have also participated in two EU projects. The past **EU-FP6 Project SEAT** focused on developing Smart technologies for stress free air travel. Within this project, we have designed a concept of local microclimate control in the area of aircraft passenger with the possibility of local temperature and relative humidity adjustment, (Zítek, Vyhlídal, et al., ***Building and Env.***, 2010, [12]). The currently running **EU-FP7 project Climate for culture** focuses on the analysis of microclimate in historical buildings under the influence of climate change. Our role in the project, in which I am a work package leader and a member of Steering Committee, is to design energy efficient non-invasive mitigation measures for microclimate control in historical interiors, following the directions set up in (Zítek, Vyhlídal, ***Building and Env.***, 2009, [13]). Next, I am also involved in the industrial project MPO TIP of Pike Automation company focused on the optimization of industrial furnaces.

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