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Metoda přesných transformací a dekompozicí nelineárních modelů a její využití v automatickém řízení

Exact transformations and decompositions of nonlinear models and their applications in automatic control

Summary

Exact transformations and decompositions of controlled dynamical systems has been intensively studied as an important part of control theory and its applications. One of the most important problems in this respect is the so-called exact feedback linearization method which enables to solve the control design for a given nonlinear system via its transformation into a simpler model, which would be at least partially linear one. In such a way it results into the decomposition of the original complex interconnected nonlinear model into a number of less complex subsystems with either simple or no connections between them. Typically, each linear part of this decomposition does not depend on the rest of the model and corresponds to some single output and single input component, while the nonlinear residuum is sufficient to be analyzed only qualitatively. Obviously, this technique significantly facilitates any further analysis and control design and therefore deserves a lot of attention. This problem has been investigated for controlled dynamical systems relatively recently, approximately starting 1970's and it possibly originated in robotics, known as the so-called computed torque principle, which is a specific case of the exact feedback linearization technique. Unlike the uncontrolled dynamical systems theory being developed since much earlier the controlled case is complicated by the presence of an additional control input variable. The brief survey of this problem will be given here, including a list of problem variations and its history. Moreover, this method will be demonstrated by the design of the underactuated walking for the simplest underactuated walking robots models. Underactuated mechanical systems are those having more degrees of freedom than actuators what complicates their control design. Nevertheless, the underactuated walking can be viewed as a more natural than the fully actuated one, as during the underactuated walking the angle between the pivot point and the pivot leg can not be directly affected by some controlled torque. For the mechanical systems, *i.e.* for the robotic models as well, there is a natural physical interpretation of linearizing transformations which makes their constructions more easy and natural. This is even more applicable for the underactuated walking where the so-called kinetic symmetry combined with a specific type of the underactuated variable enables to linearize exactly even larger part of the system than it is usual for other underatuated systems. The potential of this approach will be demonstrated by the numerical simulations for some simple underactuated devices, including the resulting walking-like movement animations.

Souhrn

Metoda přesných transformací a dekompozicí je v posledních čtyřech desetiletích intenzivně zkoumána i z pohledu teorie a aplikací automatického řízení. Jedním z důležitých problémů je zde tzv. přesná zpětnovazebná linearizace, která umožňuje řešit návrh regulace nelineárního systému prostřednictvím jeho transformace na jednodušší, alespoň částečně lineární model. Výsledkem je pak dekompozice původního nelineárního a složitě vnitřně propojeného modelu na řadu jednodušších podsystémů, buď úplně na sobě nezávislých, anebo s poměrně jednoduchým propojením. Většina podsystémů je lineární, nezávisí na ostatních a je spojena právě s jedním vybraným vstupem a jedním vybraným výstupem, zatímco nelineární residuum, jenž je jimi ovlivňováno, postačí analyzovat jen kvalitativně. Je zřejmé, že tím bude významně usnadněna další analýza a syntéza prakticky všech úloh automatické regulace, takže si metoda transformace a dekompozice nelineárních modelů zaslouží významnou pozornost. Tato metoda je pro řízené systémy teoreticky rozvíjena teprve od sedmdesátých let minulého století a má svůj praktický původ v robotice, kde byla již dříve používána v poměrně jednoduché a omezené variantě jako tzv. princip vypočteného momentu síly ("computed torque principle"). Na rozdíl od obecné teorie transformací neřízených dynamických systémů, která byla rozvíjena mnohem dříve, je teorie transformací řízených systémů komplikována přítomností vstupní proměnné. V této práci bude podán stručný přehled možných transformací i typů cílových modelů, a tím i bohaté škály různých typů přesné linearizace, včetně historie jejího výzkumu od sedmdesátých let minulého století. Dále bude tato metoda předvedena na návrhu stabilní chůze pro nejjednodušší modely podaktuovaných kráčejících robotů. Podaktuované mechanické systémy mají menší počet akčních členů, než jaký je počet jejich stupňů volnosti. Je možné říci, že podaktuovaná chůze odpovídá přirozené chůzi mnohem lépe, než plně aktuovaná, neboť moment síly působící na úhel opěrné nohy v bodě kontaktu se zemí nelze přímo ovlivnit žádným pohonem. Pro mechanické systémy, a tedy i v robotických modelech, má přesná částečná linearizace přirozenou fyzikální interpretaci a hledání příslušných transformací se tak výrazně zjednodušuje. To platí ještě více pro modely kráčení, kde tzv. kinetická symetrie v kombinaci se specifickým podaktuovaným stupněm volnosti umožňuje transformovat do lineárního tvaru ještě větší část modelu, než je tomu v případě jiných podaktuovaných systémů. Možnosti této metody budou předvedeny na numerických simulacích a animacích jednoduché chůze.

Klíčová slova: Transformace a dekompozice, automatické řízení, nelineární systémy, přesná zpětnovazebná linearizace, podaktuované mechanické systémy, kráčející robot

Keywords: Transformations and decompositions, automatic control, nonlinear systems, exact feedback linearization, underactuated mechanical systems, walking robot

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1 Introduction

Exact transformations and decompositions of nonlinear control systems constitute perhaps the most intensively studied and best understood area of the structural approach to the nonlinear control theory and systems engineering. The aim of the structural approach is to analyze the structure of the controlled dynamical systems, decompose these systems into subsystems and links between them and to use such a structural information for a better control design. Within the linear control theory it resulted into the classification of linear systems via their canonical forms where perhaps the most prominent one is the famous Brunovsky canonical form [6]. The structural theory of nonlinear systems requires more sophisticated and abstract mathematical tools as it relies on the differential geometry and notions of invariant submanifolds of differential equations. Such an approach is based on representation of the dynamical systems by vector fields of their right hand side, notions of distributions, co-distributions and conditions for their integrability known as the Frobenius and Nagano theorems, [27, 28, 30, 35, 39, 49, 56].

The practically motivated aim here is to find suitable exact compensations that leads to its equivalent and possibly the simplest representation, these representations are usually called as the canonical, normal or other forms [16, 17, 18, 40, 50, 53, 58]. The exact transformations are also usually combined with the decomposition of the systems into several subsystem [27, 28, 30, 35], where bigger part can be transformed into some well understood form while only smaller part is to be studied by more sophisticated method, specifically designed for this smaller subsystem only. Typical examples here are linear, or partially linear systems giving the so-called exact feedback linearization problem. Exact compensation then makes it possible to transfer known solutions to various control problems for these more particular and simple classes of systems to the original, seemingly more complicated models, thereby providing further efficient control design options [1, 2, 3, 4, 7, 19, 26, 38, 45, 60, 61, 62].

As a matter of fact, the exact feedback linearization principle was first introduced in robotics, known as the inverse dynamics [48] or computed torque technique [41]. Using the control theoretic terminology and more detailed definitions introduced later on, this technique is equivalent to the exact linearization via feedback transformations only, without any change of state space variables. See [37] for its exposition using robotics terminology, while survey [46] demonstrates, in particular, the broad impact of this technique on the robotic and automatic control community. In the control and systems engineering community the problem received a lot of attention in the literature and applications beginning with the pioneering works of Korobov [33], Krener [34] and Brockett [5]. Among numerous results, the key paper of Jakubzyk and Respondek [31] brought the complete differential geometric insight into the exact linearization problem in terms of distributions and vector field, while [55] gave their dual interpretation through exact one forms and co-distributions. After that, further refinements of the problem were introduced and studied: input-output linearization [29], linearization by output injection useful for observers design [36], linearization of systems with outputs [22], dynamic feedback linearization significantly improving its applicability in multi-input multi-output case [20, 21], adaptive exact feedback linearization [39], up to even more special topics, like time scaling transformations [51], topological controlled systems linearization [11], or global linearization [8, 9, 27, 50]. Surveys of these and other results can be found in papers [23, 49, 10] or in monographs [30, 32, 39, 44]. The exact feedback linearization problem is still subject of the active research even recently [57].

Nevertheless, many nonlinear systems, among them those arising in practical applications, can not be exact linearized due to the presence of singularities in possible transformations. Still, these systems can be transformed into simpler, yet nonlinear forms, that enable easier controller design than for the original nonlinear system. Example of such a situation are the so-called prime form, singular triangular forms or essentially triangular forms, [12, 13, 14, 15, 40].

The rest of the technical part of this presentation is organized as follows. The next section introduces the nonlinear model of the continuous-time controlled dynamical system and its possible transformations and decompositions. Section 3 is devoted to the exact feedback linearization method while section 4 applies the partial exact feedback linearization method to control the underactuated walking. Short final section draws some conclusions and future outlooks.

2 Nonlinear control systems, their exact transformations and decompositions

In this section, the idea of the exact system transformations and decomposition will be introduced in a more technical detail. Denote in the sequel as $C^\infty(M,N)$ the set of all smooth mappings between two smooth manifolds[1] M, N. A given smooth map is referred to as the diffeomorphism, if it is also one-to-one.

As a basic control theoretic framework, consider the following continuous-time nonlinear control system having m inputs and p outputs and described by the system of ordinary differential equations

$$\dot{x} = f(x) + G(x)u, \;\; G(x) = [g_1(x)|\dots|g_m(x)], \;\; y = (h_1(x), \dots, h_p(x))^\top, \qquad (1)$$

where $x = (x_1, \dots, x_n)^\top \in M$ is the state of the system in some local coordinate chart of a smooth n-dimensional manifold M, $u = (u_1, ..., u_m)^\top \in \mathbb{R}^m$ is its input while $y = (h_1(x), \dots, h_p(x))$ its output and $f, g_1, ..., g_m$ are smooth vector fields on M.

Many practical goals in natural and engineering systems can be described by the above framework. Typical task is to force the output of the system to behave in a desired way using the input feeded by the available information. The available information may include the measurements of the system state, but more realistically of its output only. Such an information is processed and then feeded into the input channels represented by the above m-variables $u = (u_1, ..., u_m)^\top$. Such a control is called as the feedback one, or also as the closed loop control. Information processing can be either static (input depends only on the actual measurements at time of its application), or dynamic (input uses also past information about measurements). Correspondingly, one can speak about static or dynamic feedback, furthermore, depending on the extent of the above mentioned measurements one refers either to the state or output feedback.

Feedback is both the basic notion of the control theory and the instrument for practical automatic control. It formalizes a natural intuitive idea that any control influence should

[1]Smooth manifold is the set equipped with the so-called atlas of compatible coordinate charts and some topology to determine closeness of its points. Roughly saying, locally, there is no difference between general manifold and the some $\mathbb{R}^n$ space or its open subsets and usually only these manifolds will be considered here. Nevertheless, there are some important application requiring the state space being globally a more general smooth manifold than $\mathbb{R}^n$, e.g. rigid body dynamics having the part of its state space represented by the set of all three-dimensional rotations.

reflect the actual situation of the controlled object and based on it the control action is aimed to improve that situation. Feedback loop can be also used to adjust preliminary the structure of the controlled dynamical system to simplify it. Namely, the adjusting feedback loop can be then combined with the feedback solving the control task for the simplified system thereby achieving the desired feedback controller for the original more complex system. In such a way, feedback constitutes the system transformation tool as well.

Another system transformation tool is the change of the state variable. Again, it follows rather intuitive idea: for practical control purposes only inputs and outputs are firmly given, as they represent available control tools and desired control goals, respectively. Nevertheless, the state is necessary only for the mathematical model construction, therefore, many equivalent states are possible. As a consequence, the change of the state coordinates leads to another equivalent control system representation, which can be used to control it, perhaps after some straightforward adaptations.

To be more specific, consider the new state variable $\xi \in N$ from a new manifold N, related with the original state variable x of (1) via the diffeomorphism $\mathcal{D} \in C^\infty(M, N)$:

$$\mathcal{D} : V_{\xi_0} \to U_{x_0}, \quad x = \mathcal{D}(\xi). \tag{2}$$

The new state space - the smooth manifold N - has the dimension n as well. Furthemore, consider the new input variable $w = (w_1, \ldots, w_m)^\top$ related with the original input variable u used by (1) via the smooth feedback (*i.e.*, the state dependent input space transformation):

$$u = \alpha(x) + \beta(x)w, \quad \alpha(x) \in C^\infty(U_{x_0}, \mathbb{R}^m), \ \beta(x) \in C^\infty(U_{x_0}, Gl(m, \mathbb{R})). \tag{3}$$

Here, $Gl(m, \mathbb{R})$ stands for the Lie group[2] of all invertible $(m \times m)$ matrices having real elements. Both these transformations are defined on some neighborhood U_{x_0} of $x_0 \in M$ and a neighborhood V_{ξ_0} of $\xi_0 = \mathcal{D}(x_0) \in N$, where x_0 is a selected working state of the original system. Then system (1) is said to be locally around x_0 transformed using the state transformation (2) and the feedback (3) into the following system

$$\dot{\xi} = \tilde{f}(\xi) + \tilde{G}(\xi)w, \quad y = (\tilde{h}_1(\xi), \ldots, \tilde{h}_p(\xi))^\top, \tag{4}$$

$$\tilde{f} = [\frac{\partial \mathcal{D}(\xi)}{\partial \xi}]^{-1}[f(\mathcal{D}(\xi)) + G(\mathcal{D}(\xi))\alpha(\mathcal{D}(\xi))], \quad \tilde{G} = [\frac{\partial \mathcal{D}(\xi)}{\partial \xi}]^{-1} G(\mathcal{D}(\xi))\beta(\mathcal{D}(\xi)), \tag{5}$$

$$\tilde{h}_i = (h_i(\mathcal{D}(\xi)), \ i = 1, \ldots, p. \tag{6}$$

given locally around $\xi_0 = \mathcal{D}(x_0) \in N$. Alternatively, both (1) and (4-6) are called as the **state and feedback equivalent** systems. If both above neighborhoods coincide with M and N, respectively, the equivalence is called as the global one.

System (1) is said to be (locally or globally) **decomposable** if there are the (local or global) state space transformation (2) and the feedback (3) such that the transformed system (4-6) on N, possibly after renumbering its outputs, takes the following form

$$\begin{aligned}
&\dot{\xi}^1 = f^1(\xi^1) + G^1(\xi^1)w^1, \quad y^1 = h^1(\xi^1), \ \xi^1 \in \mathbb{R}^{n_1}, \ w^1 \in \mathbb{R}^{m_1}, \ y^1 \in \mathbb{R}^{p_1} \\
&\dot{\xi}^2 = f^2(\xi) \ + G^2(\xi)w, \qquad y^2 = h^2(\xi), \ \ \xi^2 \in \mathbb{R}^{n_2}, \ w^2 \in \mathbb{R}^{m_2}, \ y^2 \in \mathbb{R}^{p_2}, \\
&G^1(\xi^1) = [g_1^1(\xi^1)| \ldots |g_{m_1}^1(\xi^1)], \quad G^2(\xi^1, \xi^2) = [g_1^2(\xi^1, \xi^2)| \ldots |g_m^2(\xi^1, \xi^2)], \\
&n_1 + n_2 = n, \quad m_1 + m_2 = m, \quad p_1 + p_2 = p, \ \ y = [y^1, y^2], \ \ \tilde{h} = [h^1, h^2]^\top, \\
&w = \begin{bmatrix} w^1 \\ w^2 \end{bmatrix}, \ \xi = \begin{bmatrix} \xi^1 \\ \xi^2 \end{bmatrix}, \ \tilde{f} = \begin{bmatrix} f^1 \\ f^2 \end{bmatrix}, \ \tilde{G} = \begin{bmatrix} g_1^1| \ldots |g_{m_1}^1| & 0 & | \ldots | & 0 \\ g_1^2| \ldots |g_{m_1}^2| & g_{m_1+1}^2 & | \ldots | & g_m^2 \end{bmatrix}.
\end{aligned} \tag{7}$$

[2]Lie group is a manifold equipped, in addition to manifold properties, by the group structure with group operation being a smooth map. For $Gl(m, \mathbb{R})$, the group operation is the usual matrix multiplication.

For the local decomposition, ξ represents some suitable coordinates chart on N, while $N = \mathbb{R}^n$ for the global decomposition. If it holds in (7) that $f^2 = f^2(\xi^2)$, $h^2 = h^2(\xi^2)$ and $g^2_{m_1+1} \equiv \ldots \equiv g^2_m \equiv 0$, than the system is said to be **decoupled** into two input-output mutually independent subsystems. It is obviously easy to define decomposition or decoupling into an arbitrary number of subsystems applying the above construction repeatedly. If $m = p$, the decoupling can go on up to the so-called complete input-output decoupling when each subsystem would correspond to some single-input and single-output (SISO) system. In such a way, complexly interconnected multi-input and multi-output (MIMO) system can be controlled as a set of independent SISO systems, what is clearly easier.

3 Exact feedback linearization of nonlinear systems

The most straightforward and natural goal for the system transformation is the linear system, either fully, or partially. If the system can be transformed into (partially) linear form using the above mentioned transformations, then it is called to be exact feedback linearizable. Of course, there are many variations of this notion, some of them will be given in detail in this section, including illustrative examples and some basic results.

Definition 1. System (1) is called locally exact feedback linearizable at $x_0 \in M$ if it is locally state and feedback equivalent via the transformation (2) and the feedback (3) into the following controllable and observable linear system

$$\dot{\xi} = F\xi + Gw, \quad y = H\xi, \quad \xi \in \mathbb{R}^n, \ w \in \mathbb{R}^m, \tag{8}$$

where F, G are $(n \times n)$, $(n \times m)$ and $(p \times n)$ matrices, respectively. The system is called restricted feedback (state) linearizable if it is feedback linearizable with $\beta(x) \equiv I_m$ ($\beta(x) \equiv I_m, \alpha(x) \equiv 0$). If, instead of the linear output relation $y = H\xi$, one has after transformation still some nonlinear relation $y = \tilde{h}(\xi)$ and linear dynamics as in (8), than the system in question is called as the system with the exact linearizable state dynamics.

Simply saying, in case of systems with the exact linearizable dynamics one need not to consider the output relation relation to be linearized as well. Situation described in Definition 1 is obviously the most desirable one, but not so realistic. Systems with the exact linearizable state dynamics being useful in many applications are better in this respect, but even that is often too much to ask. The following option, the so-called **input-output partial exact feedback linearization**, is much more realistic and therefore widely used.

Definition 2. System (1) is called locally input-output feedback linearizable at $x_0 \in M$ if it is locally state equivalent via the transformation (2) and the feedback (3) into the following partially linear system

$$\begin{aligned} &\dot{\xi}^1 = F\xi^1 + Gw, \quad y = H\xi^1, \\ &\dot{\xi}^2 = f^{nl}(\xi^1, \xi^2, w) \quad \xi^1 \in R^{n_1}, \ \xi^2 \in R^{n_2}, \ n_1 + n_2 = n, \ w \in \mathbb{R}^m, \end{aligned} \tag{9}$$

where its linear part given by matrices F, G, H is the controllable and observable one. The autonomous dynamical system without input given as

$$\dot{\xi}^2 = f^{nl}(0, \xi^2, 0) \tag{10}$$

is called as the **zero dynamics**. Nonlinear system having asymptotically stable zero dynamics is called to be the **minimum phase** one.

Notice, that the input-output linearizable system is decomposable into a fully linear controllable and observable system and some nonlinear residuum called as the zero dynamics. In general, the zero dynamics is not completely decoupled from the fully linear part and it is influenced by it. Nevertheless, it is hidden from the input output point of view, *i.e.* it does not influence the input-output linearizable part. Minimum phase property ensures that this hidden part remains stable even if it is disturbed by an exponentially decaying signal, or bounded if disturbed by bounded signal. As a consequence, for the minimum phase systems this nonlinear residuum can be ignored during the controller design [7]. Input-output linearizing transformations can be found through straightforward constructive algorithm, based on the computation of the so-called **relative degree**, [30, 44]. Moreover, the linear part may be in this case even completely input-output decoupled [28].

Obviously, for practical purposes it is desirable to have the exact systems transformations valid on largest possible domain to be applicable to all possible working regimes of the plant to be controlled. The most desirable situation in this respect is the so-called global linearization.

Definition 3. System (1) is called *globally feedback (restricted feedback, state, input-output) linearizable at* $x_0 \in M$ *to a linear system on* $\mathbb{R}^n$ if it is at this point locally linearizable and $V_0 = \mathbb{R}^n$. It is called *globally linearizable on* M if $U_{x_0} = M$. System that is linearizable globally on M to a linear system on $\mathbb{R}^n$ is called *globally linearizable.*

All types of global linearization introduced by Definition 2 are quite reasonable. Notice that for global state linearization U_{x_0} always coincides with the reachable set from x_0. Of course the case of the global linearization (*i.e.* when both $V_0 = \mathbb{R}^n$ and $U_{x_0} = M$) is the most desirable one, but at the same time it is rather restrictive one. Linearization at a given $x_0 \in M$ to a linear system on $\mathbb{R}^n$ covers cases when a nonlinear system is not globally controllable, but its restriction to the reachable set from x_0 is globally equivalent to a linear system on $\mathbb{R}^n$. This case enables a straightforward application of linear methods to solve a particular control goal for the nonlinear system. In the case of global linearization on M, one has global controllability of the original nonlinear system, while its linear equivalent is defined on an open subset of $\mathbb{R}^n$ containing the origin. Nevertheless, this case still remains better than local linearization when U_{x_0} is a proper and possibly very small subset of M.

In the sequel, where no confusion arises, various adjectives for transformations and feedbacks will be omitted

The above definitions will be illustrated by several examples. First group of examples aims to demonstrate linearization of systems dynamics, therefore the output relations will be omitted there as all claims are obviously valid for any output relation.

Example 1. The system $\dot{x}_1 = x_1 + u,\ \dot{x}_2 = u\exp(x_2)$, where $x = (x_1, x_2)^\top \in M = R^2,\ u \in \mathbb{R}$, has globally state linearizable dynamics on M but this dynamics is not globally exact feedback linearizable to a linear system on $\mathbb{R}^2$. Actually, linearizing at $x_0 = (0, a)^\top, a \in \mathbb{R}$, diffeomorphism is $x = (\xi_1 + \xi_2, -\ln(a - \xi_1))^\top$ and is defined only for $\xi_1 < a$. Nevertheless, its image is the whole $\mathbb{R}^2$.

Example 2. The system $\dot{x}_1 = x_1 + u(x_1 + x_2),\ \dot{x}_2 = ux_2$ where $x = (x_1, x_2)^\top \in M = \mathbb{R}^2,\ u \in \mathbb{R}$, has the globally state linearizable dynamics to a linear system on $\mathbb{R}^2$ at any $x_0 \in \{x \in \mathbb{R}^2 \mid x_2 \neq 0, x_1 = 0\}$, but this dynamics is not globally linearizable on M. Actually, diffeomorphism $\mathcal{D} : \mathbb{R}^2 \to \{x \in \mathbb{R}^2 \mid ax_2 > 0\},\ \mathcal{D} = (a\xi_2 + a\xi_1, a)^\top \exp \xi_1$, linearizes the system at $x_0 = (0, a)^\top$. This system is not globally controllable: reachable set from $(0, 1)^\top$ (resp. $(0, -1)^\top$) is an open halfplane $x_2 > 0$ (resp. $x_2 < 0$).

Example 3. The system $\dot{x} = f(x) + ug(x),\ x \in M = \mathbb{R}^2 \setminus \{0\},\ u \in \mathbb{R}$, where $f(x) = -(1/2)\ln(x_1^2 + x_2^2)(-x_2, x_1)^\top$, $g(x) = x$, is locally everywhere on M state linearizable, but it does not have globally linearizable in any sense of Definition 2 dynamics. Linearizing map for its dynamics is in this case

$$x = \mathcal{D}(\xi) = \exp \xi_1 \begin{bmatrix} \cos \xi_2 \\ \sin \xi_2 \end{bmatrix}, \quad \mathcal{D} : \mathbb{R}^2 \to \mathbb{R}^2 \setminus \{0\},$$

that is local diffeomorphism at any $\xi \in \mathbb{R}^2$ but is not globally invertible.

Example 4. The system on $M = \mathbb{R}^2$

$$\dot{x}_1 = \sin x_2 \cos x_2 + u \exp(-x_1) \sin x_2 \tag{11}$$

$$\dot{x}_2 = -(\sin x_2)^2 + u \exp(-x_1) \cos x_2, \tag{12}$$

is locally state linearizable everywhere, but it is not globally linearizable in any sense of Definition 2. One can easily see that the dynamics of the system (11) is transformed into a linear system defined on $\mathbb{R}^2 \setminus \{0\}$ by the smooth map $x = \mathcal{D}(\xi)$, where

$$\mathcal{D}^{-1}(x) = \exp x_1 \begin{bmatrix} \cos x_2 \\ \sin x_2 \end{bmatrix}, \quad \mathcal{D}^{-1} : \mathbb{R}^2 \to R^2 \setminus \{0\},$$

that is a local diffeomorphism, but not globally one-to-one. Considering the original nonlinear system on $\{x = (x_1, x_2)^\top \in \mathbb{R}^2 \mid x_2 \in (-\pi/2, \pi/2)\}$ and the linearized one on $\mathbb{R}^2 \setminus \{\xi = (\xi_1, \xi_2)^\top \in \mathbb{R}^2 \mid \xi_1 > 0\}$ gives the one-to-one correspondence. Note that both these sets are not invariant with respect to the corresponding systems.

Example 5. The system on $M = \mathbb{R}^2$

$$\dot{x}_1 = 2x_1 + u(1 + x_2), \qquad \dot{x}_2 = x_2 + u,$$

has globally state linearizable dynamics since its linearizing diffeomorphism $x = \mathcal{D}(\xi) = (\xi_1 + (1/2)\xi_2^2, \xi_2)^\top$ is the global diffeomorphism of $\mathbb{R}^2$ onto itself.

Previous examples concentrated on the state linearization case only, in particular, in order to illustrate briefly the main obstructions for the global linearization of systems that are locally everywhere linearizable. In Example 1 one can observe that the vector field $(1, \exp x_2)^\top$ is not **complete**, *i.e.* its integral curves are not defined for all time moments, in fact, they escape to infinity in a finite time. It will be seen in the sequel that the completeness of a certain collection of vector fields is necessary for the linearized system to be defined on the whole $\mathbb{R}^n$. Example 3 illustrates the basic topological property necessary for the global linearization: **the simple connectedness** of M that is obviously violated there. Simple connectedness usually guarantees that the linearizing diffeomorphism is globally one-to-one. Nevertheless, as indicated by Example 4, it is not true that the system, locally everywhere linearizable on a simply connected manifold M, is also globally linearizable on M.

The unrestricted feedback may provide several different linearizing transformations which makes this case even more complicated, as illustrated by the following example and results.

Example 6. Consider the following planar system with two inputs:

$$\dot{x} = \exp(-x_1) \left[(\cos x_2, -\sin x_2)^\top u_1 + (\sin x_2, \cos x_2)^\top u_2 \right], \ \forall x = [x_1, x_2]^\top \in \mathbb{R}^2.$$

Both mutually commuting vector fields on the right-hand side of this system are not complete, the system has locally state linearizable dynamics around any point, but it is not globally state linearizable in any sense of Definition 2. On the other hand, the above system has easily globally linearizable dynamics using the unrestricted feedback:

$$v = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \exp(-x_1) \begin{bmatrix} \cos x_2 & \sin x_2 \\ -\sin x_2 & \cos x_2 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}.$$

Proposition 1. Consider the planar single-input system of the form

$$\begin{aligned} \dot{x}_1 &= u \\ \dot{x}_2 &= f_0(x_2) + f_1(x_2)x_1, \quad f_0(0) = 0. \end{aligned}$$

It is

1. locally both restricted and unrestricted feedback linearizable around the origin if and only if $f_1(0) \neq 0$;
2. globally restricted feedback linearizable if and only if $f_1(x_2) \neq 0 \ \forall x_2 \in \mathbb{R}$ and $(0, f_1)^\top$ is complete vector field;
3. globally unrestricted feedback linearizable if and only if $f_1(x_2) \neq 0 \ \forall x_2 \in \mathbb{R}$.

Observe, that for the class of systems considered in Proposition 1 locally there is no difference between restricted and unrestricted feedback linearizability (of course in each case the system is linearized using different transformations). At the same time, application of the unrestricted feedback substantially enlarges possibilities for the global linearization.

Theorem 1. Single-input nonlinear system (1) has globally unrestricted feedback linearizable dynamics on M if and only if it is globally state equivalent to the following system

$$\dot{x} = f(x) + g(x)u, \ \ x = (x_1, ..., x_n)^\top \in \mathbb{R}^n,$$
$$g = (g_1(x), 0, ..., 0)^\top, \ f = (f_1(x), f_2(x), f_3(x_2, ..., x_n), ..., f_n(x_{n-1}, x_n))^\top \qquad (13)$$

and

$$g_1(x) \neq 0, \ \frac{\partial f_j}{\partial x_{j-1}} \neq 0 \ \forall x \in \mathbb{R}^n, j = 2, ..., n.$$

If in addition there exists a constant $\varepsilon > 0$ such that

$$\left|\frac{\partial f_j}{\partial x_{j-1}}\right| > \varepsilon \ \forall x \in \mathbb{R}^n, j = 2, ..., n,$$

then the above system is globally unrestricted feedback linearizable, *i.e.* the corresponding linear system is defined on the whole $\mathbb{R}^n$.

Input-output linearization can be illustrated as follows.

Example 7. Consider the system

$$\dot{x}_1 = x_1 + x_2 + x_2^3, \ \dot{x}_2 = x_2^2 + x_3, \ \dot{x}_3 = u - x_4 - x_4^5, \ \dot{x}_4 = (x_1 + x_2 + x_2^3)^2 - x_4, \ y = x_1. \quad (14)$$

The input-output linearization transformations can be found using the so-called relative degree. Roughly saying, the relative degree is the number of time derivations of the output along trajectories[3] of the system before the input explicitly appears. The corresponding

[3] Recall, that the time derivative of the function $h(x)$ along trajectories of some system $\dot{x} = f(x) + g(x)u$ is the expression $h_x(x)\dot{x} = h_x(x)(f(x) + g(x)u)$, *i.e.* the full time derivative of the time function $h(x(t))$ with $x(t)$ being a trajectory of the system $\dot{x} = f(x) + g(x)u$.

expressions then serve to define exact transformations simplifying the system structure. Namely, define ξ_1, ξ_2, ξ_3, v as the first, second, third and fourth time derivatives of the output along trajectories of (14), respectively:

$$\xi_1 := x_1,\ \xi_2 := x_1 + x_2 + x_2^3,\ \xi_3 := x_1 + x_2 + x_2^3 + (1+3x_2^2)(x_2^2 + x_3),\ v := y^{(3)} =$$

$$x_1 + x_2 + x_2^3 + (1+3x_2^2)(x_2^2+x_3) + 6x_2(x_2^2+x_3)^2 + (1+3x_2^2)(u - x_4 - x_4^5 + 2x_2(x_2^2+x_3)).$$

This means that the relative degree is equal to 3 and therefore the above algorithm determines 3 components of the new state and the feedback transformation introducing the new input variable v. To have the full coordinate change, one has to select the remaining component of the state in any way, such that the overall transformation is one-to-one, e.g. $\xi_4 := x_4$. With these new coordinates, one has the exactly transformed system representation as follows

$$\dot{\xi}_1 = \xi_2,\ \dot{\xi}_2 = \xi_3,\ \dot{\xi}_3 = v,\ \dot{\xi}_4 = -\xi_4 + \xi_2^2.$$

Therefore, the zero dynamics (see Definition 2) of the system (14) is

$$\dot{\xi}_4 = -\xi_4,$$

i.e., it is exponentially stable and system (14) is therefore the minimum phase one. This means that one can concentrate on its linear subsystem only and ignore its nonlinear one-dimensional part. In terms of decomposition terminology, the system (14) is decomposable into 3-dimensional linear controllable and observable part and the exponentially stable one-dimensional residuum influenced only by the signal ξ_2 from that linear part.

In the next section, the input-output linearization and decomposition will be demonstrated on a more practically motivated example taken from the undeactuated walking.

4 Application of exact decompositions and transformations in underactuated mechanical systems

Mechanical systems are the challenging research area with a direct path to important applications in robotics where many techniques can be directly tested and applied thanks to their physical interpretation. The well-known example in this respect is the energy as a possible source of Lyapunov function to study stability and design their control [47]. Moreover, mechanical system have a special structure as they are usually described by even number of state variables, where the first half of them are the so-called generalized coordinates while the second half are the generalized velocities. Another crucial feature is that mechanical models are usually obtained using the Euler-Lagrange formalism [25].

These specific features of the mechanical systems enable also efficient use of the presented technique of the exact transformations and decompositions. Actually, this technique is a kind of natural continuation of studies of symmetries in mechanical systems [42].

The special role is in this respect played by the so-called underacuated mechanical systems being counterpart of the fully actuated ones. Fully actuated mechanical systems are those having the same number of degrees of freedom and actuators. In this case, the exact feedback linearization technique has been widely used since long time ago. As already noted in the introduction, this technique was known in robotics even before general results on exact feedback linearization appeared and it was called as computed torque, or inverse dynamics technique [37, 41, 48].

On the other hand, in case of the **underatucated mechanical systems**, *i.e.* the systems which are not fully actuated, the exact feedback linearization is not so straightforward and only partial exact feedback linearization may be achieved. Efficient control of underactuated mechanical systems constitutes one of the most challenging problems of recent decades, see [4, 24, 43, 54, 60, 61] and references therein. Reliable and economic walking is a typical example of studies involving both control and robotic communities.

Many of these results rely on the partial feedback linearization technique combined with decomposition into blocks according to each input component. One of the simplest underactuated mechanical systems is the Acrobot. Despite being a seemingly simple system, the Acrobot comprises many important features of underactuated walking robots having degree of underactuation equal to one. Every mechanical system, including the Acrobot, straightforwardly enables certain auxiliary output having relative degree equal to two[4] which gives a two-dimensional exact feedback linearizable subsystem and a two-dimensional nonlinear zero dynamics, see e.g. [54]. Nevertheless, it turns out that specific features of the walking like underactuated systems allows a three-dimensional exact feedback linearizable part of its four-dimensional subsystem containing the non-actuated part. First, these ideas were presented in [45] and further developed in [26], but they had not been used for the control design until series of results [1, 2, 3, 19, 62]. Moreover, any walking-like mechanical system having n degrees of freedom and $n-1$ actuators can be decomposed and exactly transformed into $n-2$ two-dimensional linear systems, one three-dimensional linear system and a residual one-dimensional nonlinear dynamics. Summarizing, of total $2n$ states, $2 \times (n-2)+3 = 2n-1$ can be exactly transformed into linear models and only one dimension remains to be described nonlinearly. In other words, more general configurations can be exactly decomposed into Acrobot model and a fully actuated mechanical system which can be treated by the well-known computed torque technique. Thanks to the exact decomposition and transformation method, one can say that Acrobot comprises all peculiarity of underactuated walking and knowing how to control Acrobot walking directly opens way to the control of the general underactuated walking like configurations.

The rest of this section will be therefore devoted to the Acrobot model partial exact feedback linearization and its application to tracking of a walking-like trajectory. The Acrobot depicted on Figure 1 is a special case of an n-link chain with $n-1$ actuators attached by one of its ends to a pivot point through an unactuated rotary joint. Such a system can be modeled by the well-known Euler-Lagrange approach, see [25]. The corresponding Lagrangian is as follows

$$\mathcal{L}(q,\dot{q}) = K - V = \frac{1}{2}\dot{q}^T D(q)\dot{q} - V(q) \tag{15}$$

where q denotes an n-dimensional vector on the configuration manifold Q and $D(q)$ is the inertia matrix, K is the kinetic energy and V is the potential energy of the system. The resulting Euler-Lagrange equation is

$$\begin{bmatrix} \frac{d}{dt}\frac{\partial\mathcal{L}}{\partial\dot{q}_1} - \frac{\partial\mathcal{L}}{\partial q_1} \\ \frac{d}{dt}\frac{\partial\mathcal{L}}{\partial\dot{q}_2} - \frac{\partial\mathcal{L}}{\partial q_2} \\ \vdots \\ \frac{d}{dt}\frac{\partial\mathcal{L}}{\partial\dot{q}_n} - \frac{\partial\mathcal{L}}{\partial q_n} \end{bmatrix} = \begin{bmatrix} 0 \\ \tau_2 \\ \vdots \\ \tau_n \end{bmatrix} = \overline{u}, \tag{16}$$

[4]As the inputs are torques, any function of positions has relative degree equal to two or more.

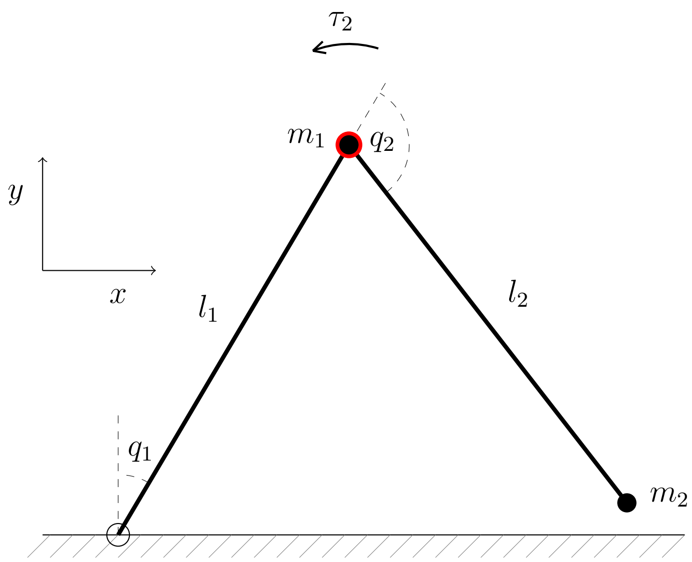

Figure 1: The Acrobot.

where $\overline{u}$ stands for the vector of external controlled forces. As already indicated, system (16) is the so-called **underactuated** mechanical system having degree of underactuation equal to one, see [54]. Moreover, the underactuated angle is the angle q_1 at the pivot point and the inertia properties of the configuration are independent of this very angle q_1. It will be shown that this combination, typical for all walking-like systems, is the key factor enabling the existence of the three-dimensional exact linearizable subsystem.

More precisely, (16) leads for the Acrobot case to a dynamic equation of the form

$$D(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = \overline{u} = [0, \tau_2]^\top \tag{17}$$

where $D(q)$ is the inertia matrix, $C(q,\dot{q})$ contains Coriolis and centrifugal terms, $G(q)$ contains gravity terms and $\overline{u}$ stands for the vector of external forces. These right hand side terms take the following particular form:

$$D(q) = \begin{bmatrix} \theta_1 + \theta_2 + 2\theta_3 \cos q_2 & \theta_2 + \theta_3 \cos q_2 \\ \theta_2 + \theta_3 \cos q_2 & \theta_2 \end{bmatrix}, \tag{18}$$

$$C(q,\dot{q}) = \begin{bmatrix} -\theta_3 \sin q_2 \dot{q}_2 & -(\dot{q}_2 + \dot{q}_1)\theta_3 \sin q_2 \\ \theta_3 \sin q_2 \dot{q}_1 & 0 \end{bmatrix}, \tag{19}$$

$$G(q) = \begin{bmatrix} -\theta_4 g \sin q_1 - \theta_5 g \sin(q_1 + q_2) \\ -\theta_5 g \sin(q_1 + q_2) \end{bmatrix}, \tag{20}$$

where the configuration vector (q_1, q_2) consists of angles defined on Fig. 1 and

$$\theta_1 = (m_1+m_2)l_1^2+I_1,\ \theta_2 = m_2 l_2^2+I_2,\ \theta_3 = m_2 l_1 l_2,\ \theta_4 = (m_1+m_2)l_1,\ \theta_5 = m_2 l_2. \tag{21}$$

The crucial property here is the above-mentioned mentioned kinetic symmetry meaning that the inertia matrix $D(q)$ depends only on the second variable q_2.

As explained earlier, the partial exact feedback linearization method is based on a system transformation into a new system of coordinates that display the linear dependence between an auxiliary output and a new input. From a theoretical point of view, the mechanical system dynamics is described by an n-dimensional state-space equation. Static state-feedback linearization using a suitable output function of relative degree r yields a linear subsystem of dimension r. In other words, the maximal feedback linearization problem consists in finding a linearizing function with maximal relative degree. In case of the Acrobot, the mentioned kinetic symmetry combined with q_1 being the underactuated angle enables to find a function $\overline{y}(q, \dot{q})$ with relative degree 3 that transforms the original system (17) by a local coordinate transformation $z = T(q, \dot{q})$ of the form

$$z_1 = \overline{y}, \quad z_2 = \dot{\overline{y}}, \quad z_3 = \ddot{\overline{y}}, \quad z_4 = f(q, \dot{q}), \tag{22}$$

into a new input/output linear system with the one-dimensional nonlinear zero dynamics:

$$\dot{z}_1 = z_2, \quad \dot{z}_2 = z_3, \quad \dot{z}_3 = \alpha(q, \dot{q})\tau_2 + \beta(q, \dot{q}) = w, \quad \dot{z}_4 = \psi_1(q, \dot{q}) + \psi_2(q, \dot{q})\tau_2. \tag{23}$$

As a matter of fact, there are two independent functions having relative degree 3 and transforming the system into the desired form (23), namely

$$\sigma = \frac{\partial \mathcal{L}}{\partial \dot{q}_1} = (\theta_1 + \theta_2 + 2\theta_3 \cos q_2)\dot{q}_1 + (\theta_2 + \theta_3 \cos q_2)\dot{q}_2, \tag{24}$$

$$p = q_1 + \frac{q_2}{2} + \frac{2\theta_2 - \theta_1 - \theta_2}{\sqrt{(\theta_1 + \theta_2)^2 - 4\theta_3^2}} \arctan\left(\sqrt{\frac{\theta_1 + \theta_2 - 2\theta_3}{\theta_1 + \theta_2 + 2\theta_3}} \tan \frac{q_2}{2}\right). \tag{25}$$

The reasons for the existence of these functions having relative degree equal to 3 have the following nice physical interpretation. Actually, by (16)

$$\dot{\sigma} = \frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{q}_1} = \frac{\partial \mathcal{L}}{\partial q_1}$$

and therefore by (15)

$$\dot{\sigma} = -\frac{\partial V(q)}{\partial q_1} = G_1(q)$$

as $D(q) \equiv D(q_2)$ by (18). In other words, $\dot{\sigma}$ has relative degree 2, *i.e.* σ has relative degree 3. Moreover, by straightforward differentiation it holds $\dot{p} = d_{11}(q_2)^{-1}\sigma$, *i.e.* $\dot{p}$ has relative degree 2, *i.e.* p should have relative degree 3 as well. Indeed, as promised earlier, in the above evaluation it is crucial, that the non-actuated angle is exactly the same one as the one with respect which there is a kinetic symmetry. At the same time, this feature is typical for the walking like movement, where the pivot point is underactuated. One can therefore expect that this type of partial exact feedback linearization of order three would play important role for underactuated walking strategies.

The zero dynamics of (23) can be used to investigate internal stability when the corresponding output is forced to be zero. For the simplest cases $\overline{y} = Cp$ or $\overline{y} = C\sigma$ the resulting zero dynamics is only critically stable. However, considering the output function $\overline{y} = C_1 p(q) + C_2\sigma(q, \dot{q})$ one gets the following zero dynamics $\dot{p} + C_1[C_2 d_{11}(q_2)]^{-1}p = 0$ which is asymptotically stable whenever C_1/C_2 is positive, $d_{11}(q_2)$ being the corresponding part of the inertia matrix D in (17). Unfortunately, the corresponding transformations have a complex set of singularities, unless C_1 is very small, which is not suitable for practical purposes.

It was shown in [19] that the above functions p, σ having maximal relative degree 3 can be used in a slightly different way. Namely, the following transformation can be defined:

$$\xi_1 = p, \quad \xi_2 = \sigma, \quad \xi_3 = \dot{\sigma}, \quad \xi_4 = \ddot{\sigma}. \tag{26}$$

Notice that by (24)-(25) and some straightforward but laborious computations the following relation holds:

$$\dot{p} = d_{11}(q_2)^{-1}\sigma, \tag{27}$$

where $d_{11}(q_2) = (\theta_1 + \theta_2 + 2\theta_3 \cos q_2)$ is the corresponding element of the inertia matrix D in (17). Applying (26), (27) to (17) gives the Acrobot dynamics in the following partial exact linearized form

$$\dot{\xi}_1 = d_{11}(q_2)^{-1}\xi_2, \quad \dot{\xi}_2 = \xi_3, \quad \dot{\xi}_3 = \xi_4, \quad \dot{\xi}_4 = \alpha(q)\tau_2 + \beta(q, \dot{q}) = w \tag{28}$$

with the new coordinates ξ and the input w being well defined whenever $\alpha(q)^{-1} \neq 0$. An important feature here is that the set of all possible transformations singularities (*i.e.*, the set where $\alpha(q)^{-1} = 0$) depends only on the position angles q_1, q_2, but not on velocities. Moreover, this set has practically favorable properties, as will be shown later on.

To determine the region where the transformation (28,26) can be applied, one has to express it explicity. Straightforward computations show that

$$\xi = \begin{bmatrix} \xi_1 \\ \xi_2 \\ \xi_3 \\ \xi_4 \end{bmatrix} = T(q_1, q_2, \dot{q}_1, \dot{q}_2) := \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \end{bmatrix}, \tag{29}$$

$$\begin{bmatrix} T_1 \\ T_3 \\ T_2 \\ T_4 \end{bmatrix} = \begin{bmatrix} p(q_1, q_2) \\ \theta_4 g \sin q_1 + \theta_5 g \sin(q_1 + q_2) \\ \Phi_2(q_1, q_2) \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix} \end{bmatrix} \tag{30}$$

where p, σ are given by (24,25) and Φ_2 by (35) later on. Furthermore, denote

$$\phi = \begin{bmatrix} \phi_1(\xi_1, \xi_3) \\ \phi_2(\xi_1, \xi_3) \end{bmatrix}, \tag{31}$$

where ϕ_1, ϕ_2 are such that

$$T_1(\phi_1(\xi_1, \xi_3), \phi_2(\xi_1, \xi_3)) = \xi_1, \qquad T_3(\phi_1(\xi_1, \xi_3), \phi_2(\xi_1, \xi_3)) = \xi_3. \tag{32}$$

It obviously holds by (29-30) that

$$\frac{\partial[\xi_1, \xi_3, \xi_2, \xi_4]^\top}{\partial[q^\top, \dot{q}^\top]^\top} = \begin{bmatrix} \Phi_1(q_1, q_2) & 0 \\ \Phi_3(q, \dot{q}) & \Phi_2(q_1, q_2) \end{bmatrix}, \quad q := [q_1, q_2]^\top, \tag{33}$$

$$\Phi_1(q_1, q_2) = \begin{bmatrix} 1 & \frac{\theta_2 + \theta_3 \cos q_2}{\theta_1 + \theta_2 + 2\theta_3 \cos q_2} \\ \theta_4 g \cos q_1 + \theta_5 g \cos(q_1 + q_2) & \theta_5 g \cos(q_1 + q_2) \end{bmatrix}, \tag{34}$$

$$\Phi_2(q_1, q_2) = \begin{bmatrix} \theta_1 + \theta_2 + 2\theta_3 \cos q_2 & \theta_2 + \theta_3 \cos q_2 \\ \theta_4 g \cos q_1 + \theta_5 g \cos(q_1 + q_2) & \theta_5 g \cos(q_1 + q_2) \end{bmatrix}, \tag{35}$$

$$\Phi_3(q,\dot q) = \left[\frac{\partial \Phi_2}{\partial q_1}\dot q \middle| \frac{\partial \Phi_2}{\partial q_2}\dot q\right] =$$

$$\left[\begin{array}{ll} 0 & -2\dot q_1\theta_3 \sin q_2 - \dot q_2\theta_3 \sin q_2 \\ -\theta_4 g\dot q_1 \sin q_1 - \theta_5 g(\dot q_1 + \dot q_2)\sin(q_1+q_2) & -\theta_5 g(\dot q_1 + \dot q_2)\sin(q_1+q_2) \end{array}\right]. \tag{36}$$

Moreover, by (31,32) it obviously holds that

$$\frac{\partial \phi(\xi_1,\xi_3)}{\partial[\xi_1,\xi_3]^\top} = \Phi_1^{-1}(q_1,q_2) = \left[\begin{array}{cc} \frac{\theta_5 g\cos(q_1+q_2)}{s(q)} & \frac{-\theta_2-\theta_3\cos q_2}{s(q)(\theta_1+\theta_2+2\theta_3\cos q_2)} \\ \frac{-\theta_4 g\cos q_1 - \theta_5 g\cos(q_1+q_2)}{s(q)} & \frac{1}{s(q)} \end{array}\right], \tag{37}$$

$$s(q) := \det\Phi_1 = g\frac{(\theta_1+\theta_3\cos q_2)\theta_5\cos(q_1+q_2) - (\theta_2+\theta_3\cos q_2)\theta_4\cos q_1}{(\theta_1+\theta_2+2\theta_3\cos q_2)}. \tag{38}$$

In other words, the coordinate change (29,30) is locally invertible at each point where

$$s(q) \neq 0. \tag{39}$$

Indeed, by (17,18) the inertia matrix $D(q) > 0$, moreover, $w = \alpha(q,\dot q)\tau_2 + \beta(q,\dot q)$ and

$$\left[\begin{array}{c} T_3 \\ w \end{array}\right] = \frac{\mathrm{d}}{\mathrm{d}t}\left[\begin{array}{c} T_2(q,\dot q) \\ T_4(q,\dot q) \end{array}\right] = \Phi_2(q)\left[\begin{array}{c} \ddot q_1 \\ \ddot q_2 \end{array}\right] + \Phi_3(q,\dot q)\dot q =$$

$$\Phi_2(q)D^{-1}(q)\left[\left[\begin{array}{c} 0 \\ \tau_2 \end{array}\right] - C(q,\dot q)\dot q - G(q)\right] + \Phi_3(q,\dot q)\dot q.$$

As a consequence, the above $\alpha(q,\dot q),\ \beta(q,\dot q)$ introduced in (28) are as follows

$$\alpha(q,\dot q) = \frac{\det\Phi_2}{\det D(q)}, \tag{40}$$

$$\beta(q,\dot q) = \frac{\det\Phi_2}{\det D(q)}(-C_2(q,\dot q)\dot q - G_2(q)) \tag{41}$$

$$-\frac{(\theta_2\theta_4 g\cos(q_1) - \theta_3\theta_5 g\cos(q_2)\cos(q_1+q_2))(C_1(q,\dot q)\dot q + G_1(q))}{\det D(q)} \tag{42}$$

$$-\theta_4 g\dot q_1^2\sin q_1 - \theta_5 g(\dot q_1+\dot q_2)^2\sin(q_1+q_2), \tag{43}$$

where Φ_2 is given by (35). By virtue of [10] and the references therein, the coordinate change (30) is globally invertible on any open set where (39) holds and which is both connected and simply connected. In other words, the Acrobot model is state and feedback equivalent to system (28) on any such set. Fig. 2 depicts some of these sets.

In the sequel one can therefore concentrate on the study of system (28). This system is almost linear, but there is a nonlinearity $d_{11}(q_2)^{-1}$ in the first row that depends on q_2 only. To keep consistently new variables, this nonlinearity should be expressed in coordinates ξ as $d_{11}^{-1}(\phi_2(\xi_1,\xi_3))$, ϕ_2 is given by (31,32). Such an expression is a quite complicated one, but one can use its certain favorable qualitative properties. Namely, straightforward computations give

$$a_{min} \leq d_{11}(q_2)^{-1} \leq a_{max}, \tag{44}$$

$$a_{min} = \frac{1}{m_2(l_1+l_2)^2 + m_1 l_1^2 + I_1 + I_2},\ a_{max} = \frac{1}{m_2(l_1-l_2)^2 + m_1 l_1^2 + I_1 + I_2}, \tag{45}$$

$$a_{max} - a_{min} = \frac{4l_1 l_2 m_2(m_2(l_1+l_2)^2 + m_1 l_1^2 + I_1 + I_2)^{-1}}{(m_2(l_1-l_2)^2 + m_1 l_1^2 + I_1 + I_2)}, \tag{46}$$

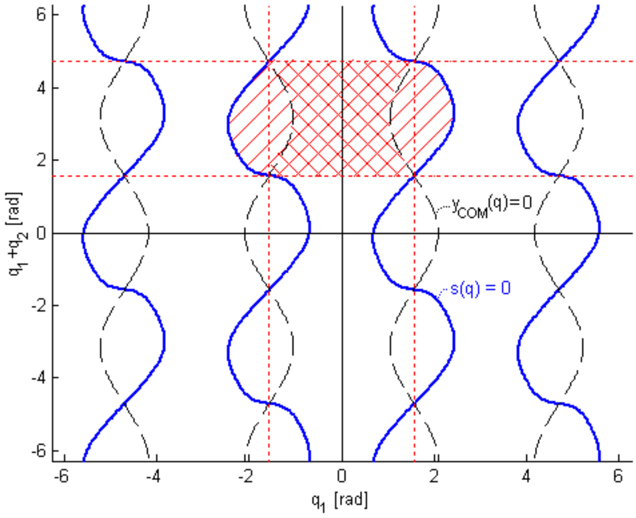

Figure 2: Singularities and possible regular set of coordinate change (30). Here $s(q)$ is given by (38), while $y_{COM}(q)$ stands for the vertical distance of the Acrobot centre of mass from the ground. Notice, that in real application this distance should obviously be significantly bigger than zero.

i.e. $a_{max} - a_{min}$ is quite small and therefore the nonlinearity $d_{11}(q_2)^{-1}$ is actually varying in a quite narrow range. Therefore, its derivative also evolves in a favorable way, namely

$$\frac{\partial[d_{11}(q_2)^{-1}]}{\partial q_2} = (2\theta_3 \sin q_2) d_{11}(q_2)^{-2}, \ |\frac{\partial[d_{11}^{-1}]}{\partial q_2}| \leq 2\theta_3 a_{max}^2. \tag{47}$$

To ensure exponential tracking of a given walking-like trajectory some additional qualitative prpoerties of $d_{11}^{-1}(\phi_2(\xi_1, \xi_3))$ should be developed. Namely, assume that an open-loop control generating a suitable single step reference trajectory is given on the time interval $[0, T]$ in partial exact linearized coordinates (28). Therefore, our task is to track the following reference system

$$\dot{\xi}_1^{ref} = d_{11}^{-1}(q_2^{ref})\xi_2^{ref}, \quad \dot{\xi}_2^{ref} = \xi_3^{ref}, \quad \dot{\xi}_3^{ref} = \xi_4^{ref}, \quad \dot{\xi}_4^{ref} = w^{ref}. \tag{48}$$

To demonstrate the strength of the exact feedback transformation technique by analyzing partially linear form (28). Namely, subtracting (48) from (28) one has (put $e := \xi - \xi^{ref}$):

$$\dot{e}_1 = d_{11}^{-1}(\phi_2(\xi_1, \xi_3))\xi_2 - d_{11}^{-1}(\phi_2(\xi_1^{ref}, \xi_3^{ref}))\xi_2^{ref}, \dot{e}_2 = e_3, \ \dot{e}_3 = e_4, \ \dot{e}_4 = w - w^{ref}.$$

Straightforward computations based on Taylor expansions give

$$\dot{e}_1 = \mu_2(t)e_2 + \mu_1(t)e_1 + \mu_3(t)e_3 + o(e) \tag{49}$$

$$\dot{e}_2 = e_3, \quad \dot{e}_3 = e_4,, \quad \dot{e}_4 = w, \tag{50}$$

$$\mu_1(t) = \xi_2^{ref}(t)\frac{\partial[d_{11}^{-1}]}{\partial q_2}\frac{\partial\phi_2}{\partial\xi_1}(q_2^{ref}(t)), \ \ \mu_2(t) = d_{11}^{-1}(q_2^{ref}(t)), \tag{51}$$

$$\mu_3(t) = \xi_2^{ref}(t)\frac{\partial[d_{11}^{-1}]}{\partial q_2}\frac{\partial\phi_2}{\partial\xi_3}(q_2^{ref}(t)), \quad q_2^{ref}(t) = \phi_2(\xi_1^{ref}(t), \xi_3^{ref}(t)), \ q_2 \in [0, 2\pi). \tag{52}$$

By (44-47) for every walking-like step there are some constants $\mathcal{B}, \mathcal{R} > 0$ such that

$$|\mu_1(t)| \leq 2\theta_3 a_{max}^2(\theta_4 + \theta_5)\frac{\mathcal{R}}{\mathcal{B}}, \tag{53}$$

$$|\mu_3(t)| \leq 2\theta_3 a_{max}^2 \frac{\mathcal{R}}{\mathcal{B}}, \;\; 0 < a_{min} \leq \mu_2(t) \leq a_{max}. \tag{54}$$

Constants $\mathcal{B}, \mathcal{R}$ characterize minimal distance from transformations singularities and maximal velocity of walking. So, closer to singularity and faster to walk would create more difficulties in design due to higher values of $\mu_3(t)$. Singularities correspond to points where Acrobot is difficult to control from basic mechanical reasons while faster walking is obviously a more challenging task. Therefore, the above dependence might have been intuitively expected. More precisely, these constants are defined as follows

$$\forall t \geq 0 \;\; |s(\phi_2(\xi^{ref})(t))| \geq \mathcal{B} > 0, \;\; |\xi_2^{ref}(t)| \leq \mathcal{R}, \; \forall t \geq 0, \tag{55}$$

where ϕ_2 is given by (31,32) and $s(q)$ by (38). Moreover, it turns out that for any given reference trajectory $q^{ref}(t)$, the functions $\mu_{1,2,3}$ can be quite easily computed numerically using formulas (51-52). Summarizing, one has to stabilize the linear time-varying system (49-50) using a linear feedback. One option is to use the quadratic stability concept that would ensure the existence of a single linear feedback and a single quadratic Lyapunov function for all possible values of the three-dimensional parameter $[\mu_1(t), \mu_2(t), \mu_3(t)], \; t \in [0, T]$ where $T > 0$ is the time duration of a single step reference trajectory.

To perform this plan define the state $x(t) = e(t)$ as the error signal and consider the following open-loop continuous time-varying linear system

$$\dot{x}(t) = A(t)x(t) + Bu(t), \quad A(t) = \begin{pmatrix} \mu_1(t) & \mu_2(t) & \mu_3(t) & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \;\; B = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}. \tag{56}$$

The tracking problem consists in finding the state-feedback controller $u(t) = Kx(t), \;\; K = [K_1 \; K_2 \; K_3 \; K_4]$, producing the following exponentially stable closed-loop system

$$\dot{x} = (A + BK)\, x = \begin{pmatrix} \mu_1(t) & \mu_2(t) & \mu_3(t) & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ K_1 & K_2 & K_3 & K_4 \end{pmatrix} x, \tag{57}$$

where bounds for $\mu(t) = (\mu_1(t), \mu_2(t), \mu_3(t))$ are given by (53)-(54).

Despite entries of $\mu(t)$ are **known** functions, the appealing idea is to treat them as **unknown disturbances** satisfying the above mentioned given constraints. If constraints are tight enough, one can think about solving quadratic stability conditions and design a unique feedback stabilizing such an "uncertain" system. Obviously, such a feedback would be at the same time solving our tracking problem.

To pursue such an idea, one can obtain LMI conditions for the quadratic stability as follows. Recall here that the quadratic stability is a particular case of robust stability, valid for arbitrarily fast time-variation of the uncertain parameters and certified by a unique quadratic-in-the-state parameter-independent Lyapunov function. Consider the well-known Lyapunov inequality to be solved for all values of $\mu(t)$ by finding a suitable symmetric positive definite matrix S and a vector K:

$$(A(\mu) + BK)^{\mathrm{T}} S + S (A(\mu) + BK) \preceq 0, \quad S = S^{\mathrm{T}} \succ 0. \tag{58}$$

Such a problem is in fact bilinear with respect to the unknowns S, K. Denoting

$$Q = S^{-1}, \; Y = KS^{-1} \tag{59}$$

gives the following LMI condition for quadratically stabilizing feedback design:

$$A(\mu)Q + BY + (A(\mu)Q + BY)^T \preceq 0, \quad Q \succ 0, \tag{60}$$

see e.g. [52, Section 5.2]. Notice that pair $(A(\mu), B)$ is controllable if and only if

$$\mu_1\mu_3 + \mu_2 \neq 0. \tag{61}$$

Obviously, if the set of possible values of μ contains, or stays close to, the singular set given by (61), LMI (60) becomes infeasible, or almost infeasible.

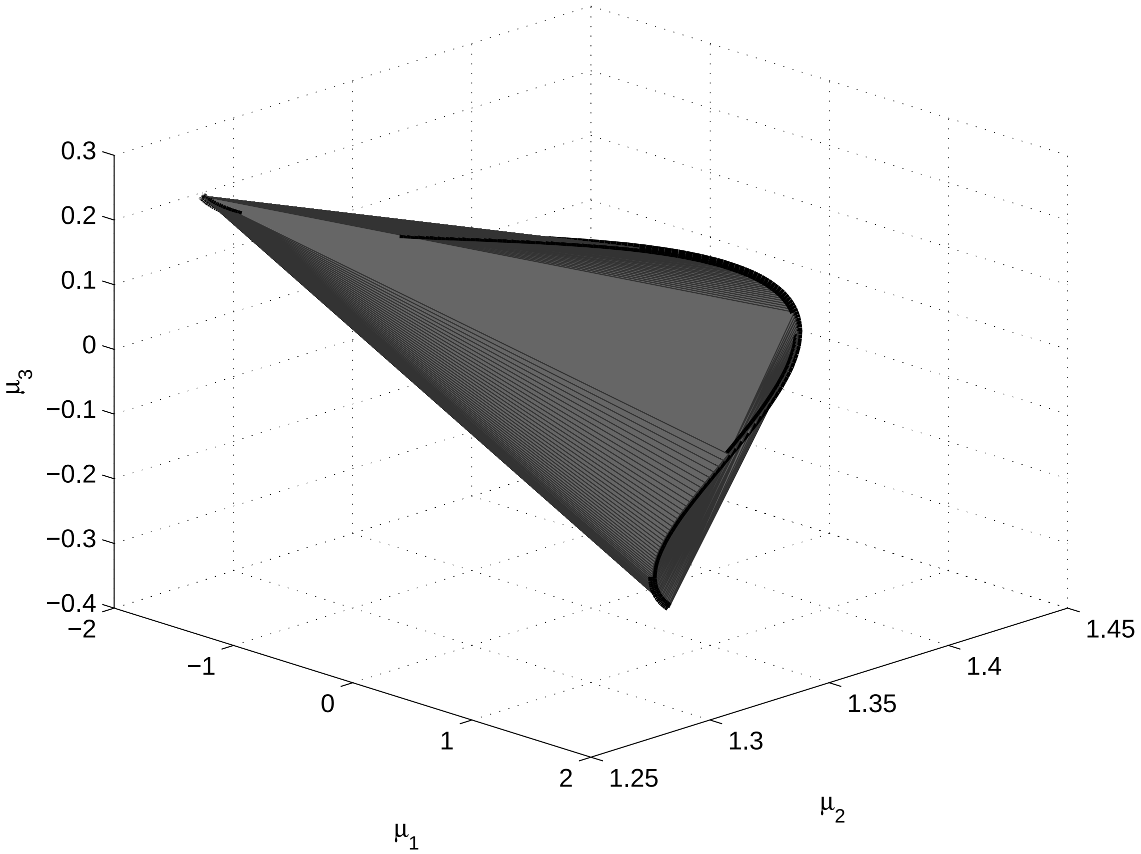

Figure 3: Trajectory $\mu(t)$ and its convex hull.

As already indicated, values of $\mu(t)$ during a given single step can be computed numerically. A detailed account of approximate modeling of the $\mu(t)$ trajectory and corresponding LMI conditions used to generate a stabilizing feedback gain can be provided as follows.

To demonstrate the above approach to tracking feedback design, the so-called pseudo-passive walking trajectory, developed in [19] can be used. Briefly, the pseudo-passive walking trajectory is the one which is produced by the zero virtual input w, *i.e.* by the real torque $\tau_2 = -\beta/\alpha$, where α, β are given by (28). By physical considerations it means that the pseudo-passive trajectory maintains the constant speed of the center of mass of the whole Acrobot. For such a trajectory, the time varying entries $\mu_{1,2,3}(t)$ were computed numerically with high precision using the same Acrobot physical parameters as in [19]. In [2], these entries were embedded in various kind of convex polytopic sets and the corresponding LMI problems were solved, thereby obtaining the quadratic stability of the error dynamics with various degrees of conservatism. The corresponding results were thoroughly compared in numerical experiments and simulations.

The best results were achieved in [2] as follows. It was chosen to sample the trajectory at time instants t_i, and to let $A_i = A(\mu(t_i))$ for $i = 1, \ldots, N$. The corresponding uncertainty model is the polytopic convex hull of the A_i vertices. Taking $N = 279$ equidistant time instants, the resulting convex hull is a polytope with 274 vertices and 544 facets in the parameter space. Even though it is not guaranteed that the genuine trajectory $\mu(t)$ is contained in this polytope, it is very close to the actual convex hull of the trajectory. The convex hull can be seen on Fig. 3. Solving the H_2 design LMIs (see [2] for more details) the state-feedback matrix is obtained $K = 10^3 \cdot (-3.3407 - 2.0073 - 0.29683 - 0.024386)$ having the Euclidean norm $3.9087 \cdot 10^3$.

Applying the feedback computed based on these gains and transformed to original coordinates one obtains asymptotically stable tracking of the pseudo-passive walking trajectory. To illustrate this approach more transparently, Figure 4 shows the animation of the Acrobot walking-like single-step trajectory with the above state-feedback gain matrix torque saturation of $\pm 10\mathrm{Nm}$. These animation shows that the above computed strategy has a nice intuitive interpretation: to make up the missing speed of the pivot angle, which is underactuated, the Acrobot speeds and then brakes the swing leg, thereby creating the missing torque at the pivot point.

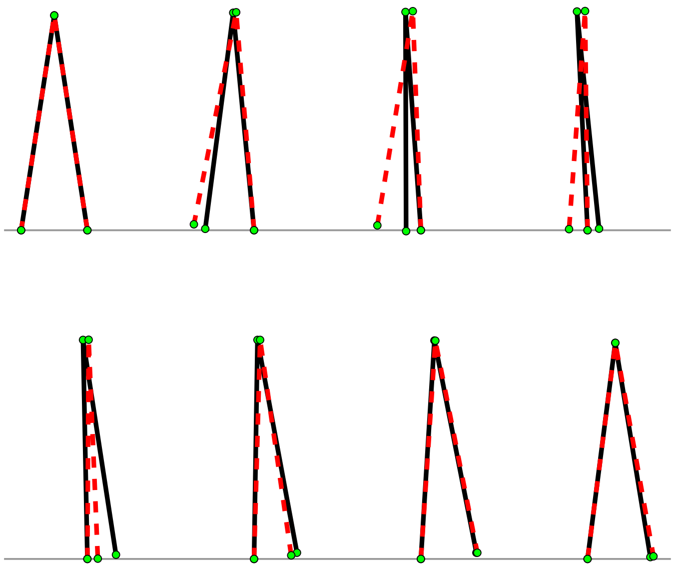

Figure 4: Animation of a single step with sampling time $0.08\,\mathrm{s}$. The dashed line is the reference, the full line represents the controlled Acrobot model.

5 Conclusions

The method of the exact transformations and decompositions of nonlinear controlled dynamical systems has been presented, including its theoretical basics and history of its research and applications development. This method has been demonstrated in detail on the appealing problem of the underactuated walking design for the simplest walking-like mechanical systems known as the Acrobot. These results has future potential to be extended to any reasonable general underactuated walking-like configurations via their special decomposition into a fully actuated system and some virtual Acrobot-like model. As a consequence, the control for general system would be a straightforward combination of the

Acrobot control and the well-known computed torque technique. This new idea is currently a subject of an intensive research of the author, his colleagues and students.

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Expertise

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Scientific societies and editorial boards

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1999-2000 CINVESTAV Guadalajara, Mexico, ”Modern Nonlinear Control” and ”Advanced Nonlinear Control”, 120 hours in total. **2003-2010** Faculty of Electrical Engineering, Czech Technical University in Prague, ”Nonlinear Systems”, master level courses and PhD level courses, 338 hours in total. **2002-2010** Supervisor of 4 successfully defended PhD students. Currently advisor of 3 PhD students. **From 2001** Member of Study Branch Board for PhD study branches ”Control Technology and Robotics”. **From 2009** Member of Study Branch Boards for PhD study branches ”Measurement Technology” and ”Control and Operation of Air Traffic”.

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