České vysoké učení technické v Praze, Fakulta elektrotechnická

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Analytický návrh stejnoměrně zvlněných IIR filtrů

Analytical Design of IIR Equiripple Filters

Summary

This lecture presents a novel approach in the analytical design of IIR complementary filter pair. The analytical solution in the z domain is presented for an IIR complementary filter pair which exhibits optimum equiripple behaviour of the frequency response over both bands. The solution gives a direct decomposition of the transfer function into two all-passes and provides an easy access to the design formulae for lattice wave digital filters refraining from the continuos-time counterparts. The multiplier coefficients are obtained in a simple algebraic form comprising the zeros and real pole of the transfer function only. The bireciprocal case represented by the transfer function with purely imaginary poles is also included. Design examples and computer simulations of wave digital lattices are presented.

Souhrn

Přednáška je zaměřena na nový přístup při analytickém návrhu IIR komplementární dvojice filtrů. Analytické řešení v z-rovině je prezentováno na IIR komplementární dvojici filtrů, které vykazují optimální stejnoměrné zvlnění kmitočtové charakteristiky filtru v obou pásmech. Řešení vede na přímou dekompozici přenosové funkce na dva fázovací články a poskytuje snadný přístup k nalezení návrhových vztahů pro vlnové číslicové filtry na bázi analogového prototypu filtru ve tvaru křížového článku. Koeficienty násobiček jsou získány v jednoduchém algebraickém tvaru z nul a reálných pólů přenosové funkce. Je prezentován i bireciproký typ filtru charakterizovaný přenosovou funkcí s čistě imaginárními póly. Na závěr jsou uvedeny příklady návrhu vlnových číslicových filtrů ve tvaru kříže a jejich simulace na počítači.

Klíčová slova : vlnový číslicový filtr, křížový filtr, náavrh filtru, stejnoměerně zvlněný pár

Key words : Wave Digital Filter, Lattice, Filter Design, Equiripple Filter Pairs

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1. Introduction

The all-pass decomposition of a design transfer function plays a fundamental role in the design of IIR complementary filters [1]. Each of the two parallel sections formed by an all-pass can be implemented in a variety of ways, either by a simple cascade of first and second order all-pass sections [6] or by using the Gray and Markel procedure [5] or, alternatively, in the form of a wave digital lattice [4]. While the Gray and Markel structures are designed starting from a discrete-time transfer function from which an all-pass function is built up with the same denominator, the wave digital filters are usually obtained from analog filters by bilinear transformation inheriting the low sensitivity property of the lossless LC prototypes [2]. It is the purpose of this paper to extend the design method [10] for a pair of equiripple IIR complementary filters suitable for the all-pass decomposition and to develop an all-pass section synthesis without a recourse to the continuous-time domain. The closed form solution of the equiripple IIR filter approximation is confined to the symmetrical w-domain to which arbitrary nonsymmetrical filter specifications are to be transformed. In contrast to the bilinear transformation method of an analog elliptic filter[4] the direct z-domain, w-domain or, respectively, approximation to the minimax requirements provides a wider family of transfer functions [2], [10] which can be realized as a parallel connection of two all-pass sections. For example, the explicit minimax solution is obtained through the constrained design of complementary pairs in the symmetrical w-domain yielding the odd degree transfer function with the degree of the denominator less than the degree of the numerator by one. In this design the low-pass and high-pass magnitudes are controlled by the position of the passband edges. The symmetrical w-domain specifications are accomplished after the following chain of low-pass to low-pass transformations

$$\begin{aligned} \overline{z} \longrightarrow \overline{w} = \frac{1}{2}(\overline{z} + \frac{1}{\overline{z}}) \quad &\longrightarrow \quad w = \frac{\overline{w} - a}{1 - a\overline{w}} \\ &\longrightarrow \quad z = w + \sqrt{w^2 - 1} \\ &\longrightarrow \quad \overline{z} = \frac{z + b}{1 + bz} \end{aligned} \tag{1}$$

where

$$a = \frac{1 + \overline{w}_p \cdot \overline{w}_s - \sqrt{1 - \overline{w}_p^2} \cdot \sqrt{1 - \overline{w}_s^{\,2}}}{\overline{w}_p + \overline{w}_s} \tag{2}$$

$$b = \frac{1 - \sqrt{1 - a^2}}{a} \tag{3}$$

and for a given passband and stopband edges ω_p, ω_s and sample frequency ω_0

$$\overline{w}_p = \cos \omega_p T, \qquad \overline{w}_s = \cos \omega_s T, \qquad T = \frac{2\pi}{\omega_0} \ . \tag{4}$$

A main part of this paper is devoted to the w-z domains where the actual design process runs. In section 2 the design formulae for the equiripple approximation of IIR digital filters and complementary filter pairs in the w-domain are reviewed. The results are extended to the z-domain. In section 3 the all-pass decomposition is closely related to the equiripple filter design itself and the distribution of the stable poles is obtained from the factorization of the characteristic equation. In section 4 the design steps for lattice wave digital filters are summarized in the form of a cookbook. In section 5 several design examples are presented and numerical simulations of wave lattices are performed.

2. Solution of the equiriple approximation in the z-domain

The design procedures for both a pair of complementary filters or a single filter with equiripple frequency response were developed in [10],[11]. Here we concisely review these formulae and often refer to the references mentioned. From the given filter specifications, e.g. the passband and stopband edges, ω_p, ω_s, the sampling frequency ω_0 and the corresponding attenuation levels a_p, a_s we form the discrimination factor

$$k_1 = \sqrt{\frac{10^{0.1a_p} - 1}{10^{0.1a_s} - 1}} \ , \tag{5}$$

the passband ripple

$$\epsilon = \sqrt{10^{0.1a_p} - 1} \tag{6}$$

and using (2), (4) the selectivity factor

$$\pm k' = \frac{\overline{w}_{p,s} - a}{1 - a \cdot \overline{w}_{p,s}} \tag{7}$$

which are central parameters of the approximation procedure (cf. Fig. 1). The low-pass magnitude is designed in the symemtrical w-domain using the characteristic equation

$$H(z)H(z^{-1}) = Q(w) = \frac{1}{1 + \epsilon^2 F^2(w)} \ . \tag{8}$$

In the following the moduli of the elliptic functions k_1' and k' fulfil $k_1^2 + {k_1'}^2 = 1$ and $k^2 + {k'}^2 = 1$. The equiripple solution is described by a pair of Jacobian elliptic functions in parametric form. The rational form of the characteristic function $F^2(w)$ is obtained by introducing the frecuency mapping $w = dn(u \mid k)$ for the integer solution n of the degree equation (11)

$$2n \frac{\mathbf{K}(k')}{\mathbf{K}(k)} = \frac{\mathbf{K}(k_1')}{\mathbf{K}(k_1)} \tag{9}$$

where $\mathbf{K}$ denotes the complete elliptic integral. The odd degree solution $n = 2m+1$ leads to

$$F^2(w) = \frac{1}{k_1} \cdot \frac{1-w}{1+w} \cdot \prod_{\mu=1}^{m} \Big[\frac{w - w_{0\mu}}{w + w_{0\mu}}\Big]^2 \tag{10}$$

with $w_{0\mu} = dn(\frac{2\mu}{n}\mathbf{K}(k) \mid k)$ for $\mu = 1, 2, \dots, m$, while for the even degree solution $n = 2m$ we get

$$F^2(w) = \frac{1}{k_1} \cdot \prod_{\mu=1}^{m} \Big[\frac{w - w_{0\mu}}{w + w_{0\mu}}\Big]^2 \tag{11}$$

with $w_{0\mu} = dn(\frac{2\mu - 1}{n}\mathbf{K}(k) \mid k)$ for $\mu = 1, 2, \dots, m$. The poles of the transfer function are found from (8) which provides the following equation in parametric form

$$1 + \epsilon^2 F^2(w)\Big|_{w=dn(u|k)} = 1 + \epsilon^2 sn^2\Big(n\frac{\mathbf{K}(k_1)}{\mathbf{K}(k)}u \mid k_1\Big) = 0 \ . \tag{12}$$

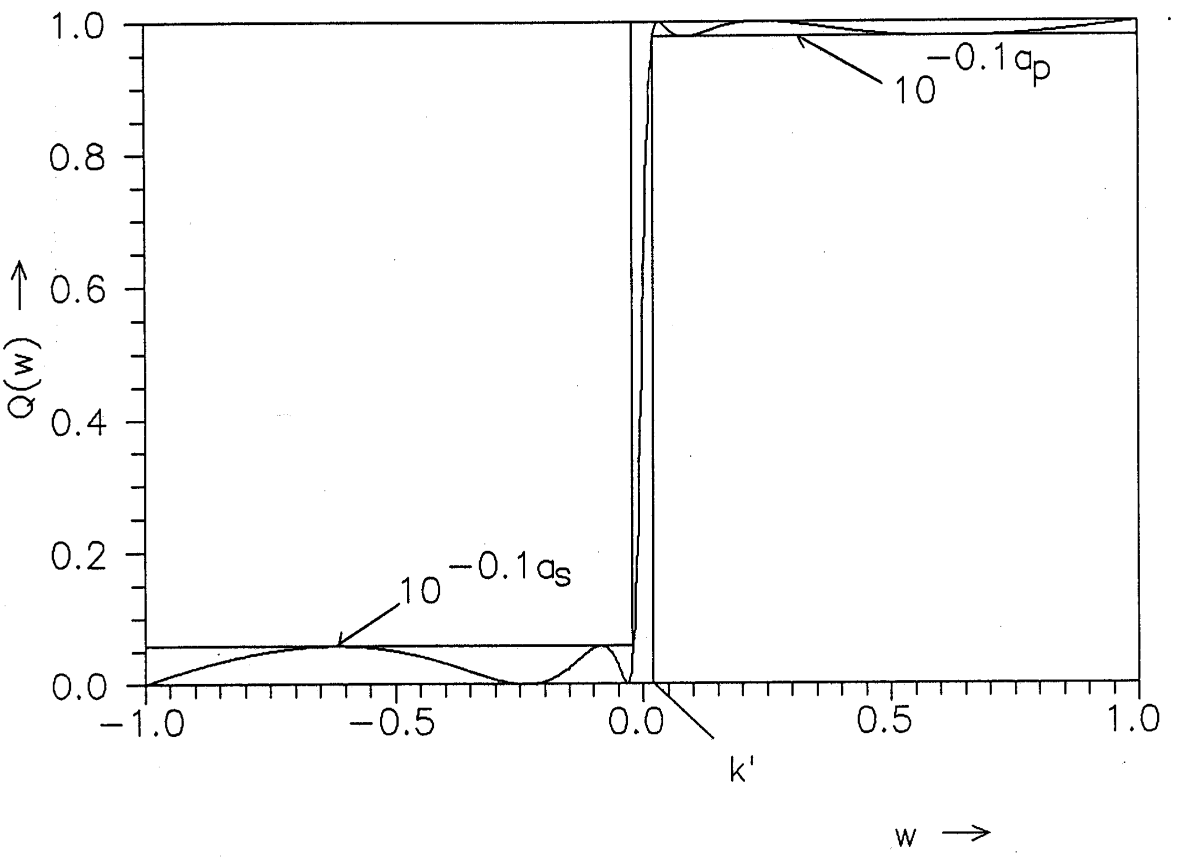

Figure 1: Equiripple approximation in the w-domain.

Both the odd and the even degree poles are formally identical except of the real pole which in the even degree solution plays the role of an auxiliary parameter. Let $w_\mu = \alpha_\mu \pm j\beta_\mu$, then

$$\alpha_0 = dc(v_\infty \mid k'), v_\infty = \frac{\mathbf{K}(k)}{n\mathbf{K}(k_1)} F(arctan\frac{1}{\epsilon} \mid k_1') \tag{13}$$

$$\alpha_\mu = \frac{k^2 \cdot \alpha_0 \cdot w_{0\mu}}{k^2 + (1 - w_{0\mu}^2)(\alpha_0^2 - 1)} \tag{14}$$

$$\beta_\mu = \frac{\sqrt{1 - w_{0\mu}^2}\sqrt{w_{0\mu}^2 - k'^2}\sqrt{\alpha_0^2 - 1}\sqrt{w_0^2 - k'^2}}{k^2 + (1 - w_{0\mu}^2)(\alpha_0^2 - 1)} \tag{15}$$

Corresponding z-domain formulae are expressed using basic interrelations among Jacobian elliptic functions [1]. This approach avoids finding the square roots of complex numbers and enables us to recognize those poles which are within the unit circle in the z-domain. It is then preferable to transform first from the symmetrical w-domain to the symmetrical z-domain before the results are transformed back to the original $\overline{w}$ and $\overline{z}$ domains. The w- and z-domains are connected by

$$z = w + \sqrt{w^2 - 1} = dn(u \mid k) + \mathrm{j}k \cdot sn(u \mid k) \tag{16}$$

which gives the zeros of the characteristic function as

$$z_{o\mu} = dn(\frac{2\mu}{n}\mathbf{K}(k) \mid k) + \mathrm{j}\ \mathrm{k} \cdot sn(\frac{2\mu}{n}\mathbf{K}(k) \mid k) \tag{17}$$

for $n = 2m+1$,$\mu = 1, \ldots, m$ and similarly

$$z_{o\mu} = dn(\frac{2\mu - 1}{n}\mathbf{K}(k) \mid k) + \mathrm{j}\ \mathrm{k} \cdot sn(\frac{2\mu - 1}{n}\mathbf{K}(k) \mid k) \tag{18}$$

for $n = 2m, \mu = 1, \ldots, m$. The properties of the elliptic functions give intrinsically

$$z_{0\mu} \cdot z^*_{0\mu} = dn^2(x \mid k) + k^2 sn^2(x \mid k) = 1 \tag{19}$$

where x stands for $(2\mu/n)\mathbf{K}(k)$, $[(2\mu - 1)/n]\mathbf{K}(k)$ respectively. The poles of the transfer function are deduced from (16) as

$$\begin{aligned} z_\mu &= dn(\pm jv_\infty + x \mid k) + \mathrm{j}\ \mathrm{k}\ sn(\pm jv_\infty + x \mid k) \qquad (20) \\ &= \Big[dn(x \mid k)cn(v_\infty \mid k') + \mathrm{j}\ \mathrm{k}\ sn(x \mid k\Big] \frac{dn(v_\infty \mid k') \pm kcn(x \mid k)sn(v_\infty \mid k')}{cn^2(v_\infty \mid k') + k^2 sn^2(x \mid k) sn^2(v_\infty \mid k')} \ . \end{aligned}$$

Let $z_\mu = a_\mu \pm \mathrm{j} b_\mu$, then

$$a_\mu = \Re(z_\mu) = kw_{0\mu} \frac{k\alpha_0 \pm \sqrt{\alpha_0^2 - 1}\sqrt{w_{0\mu}^2 - k'^2}}{k^2 + (1 - w_{0\mu}^2)(\alpha_0^2 - 1)} \tag{21}$$

$$b_\mu = \Im(z_\mu) = \sqrt{1 - w_{0\mu}^2}\sqrt{\alpha_0^2 - k'^2}\ \frac{k\alpha_0 \pm \sqrt{\alpha_0^2 - 1}\sqrt{w_{0\mu}^2 - k'^2}}{k^2 + (1 - w_{0\mu}^2)(\alpha_0^2 - 1)} \ . \tag{22}$$

The poles within the unit circle are those with the minus sign in (21), (22). Since the identities

$$k^2 w_{0\mu}^2 + (1 - w_{0\mu}^2)(\alpha_0^2 - k'^2) = k^2 + (1 - w_{0\mu}^2)(\alpha_0^2 - 1) = k^2\alpha_0^2 - (\alpha_0^2 - 1)(w_{0\mu}^2 - k'^2) \tag{23}$$

hold we finally obtain

$$a_\mu = \frac{k \cdot w_{0\mu}}{k\alpha_0 + \sqrt{\alpha_0^2 - 1}\sqrt{w_{0\mu}^2 - k'^2}} \ , \quad b_\mu = \frac{\sqrt{1 - w_{0\mu}^2}\sqrt{\alpha_0^2 - k'^2}}{k\alpha_0 + \sqrt{\alpha_0^2 - 1}\sqrt{w_{0\mu}^2 - k'^2}} \tag{24}$$

and the relation

$$a_\mu^2 + b_\mu^2 = \frac{k\alpha_0 - \sqrt{\alpha_0^2 - 1}\sqrt{w_{0\mu}^2 - k'^2}}{k\alpha_0 + \sqrt{\alpha_0^2 - 1}\sqrt{w_{0\mu}^2 - k'^2}} < 1 \tag{25}$$

which ensures the stability. The re-transformation back to the original $\overline{z}$-domain preserves stability if from (2) , (3) $\mid a \mid < 1$ holds. The real pole is transformed into the z-domain by

$$a_0 = \alpha_0 - \sqrt{\alpha_0^2 - 1} \ . \tag{26}$$

Though these results are generally valid, in the following, we refrain from the even degree solutions since the odd degree transfer functions only permit an all-pass decomposition with real coefficientes and therefore they are used in the design of lattice wave digital filters. The constrained design of an equiripple complementary pair is accomplished if we, before evaluating the poles from (12), impose

$$\frac{\epsilon^2}{k_1} = \frac{1-a}{1+a}\prod_{\mu=1}^{m}\Big[\frac{1-aw_{0\mu}}{1+aw_{0\mu}}\Big]^2 \tag{27}$$

as an additional constraint upon the design parameters ϵ and k_1. In view if the inverted degree equation (9), (10)

$$k_1 = \frac{1-k'}{1+k'}\prod_{\mu=1}^{m}\Big[\frac{k'-w_{0\mu}}{k'+w_{0\mu}}\Big]^2 \tag{28}$$

it turns out that in the complementary filter design the frequency and the amplitude specifications are not by any means independent. From k_1 and ϵ^2 connected by (27) and (28) with the frecuency specifications, the available attenuations in the passband and the stopband are obtained as

$$a_p = 10log(1+\epsilon^2)\ , \qquad a_s = 10log(1+\frac{\epsilon^2}{k_1^2})\ . \tag{29}$$

In the symmetrical w-domain the complementary constraint (27) forces the real pole α_0 to be negative reciprocal to a, e.g.

$$\alpha_0 = -\frac{1}{a}\ . \tag{30}$$

The re-transforming back to the original nonsymmetrical $\overline{w}$-domain the real pole $\overline{\alpha_0}$ is shifted to infinity (while in the $\overline{z}$-domain the real pole $\overline{a_0}$ is moved to the origin) leaving the order of the denominator of the transfer function decreased by one. presented. The so-called bireciprocal case is obtained from the previous results reducing them to the halfband limit, e.g.

$$\overline{w}_p = -\overline{w}_s = k'\ . \tag{31}$$

Consequently, we get from (1), (27), (29) a set of elementary conditions

$$a = 0, \quad \epsilon^2 = k_1, \quad 10^{-0.1a_p} + 10^{-0.1a_s} = 1 \tag{32}$$

which finally lead to the low-pass magnitude

$$Q(w) = Q\cdot(1+w)\prod_{\mu=1}^{m}\frac{(w+w_{0\mu})^2}{w^2+\beta_\mu^2}\ , \qquad Q = \frac{1}{2\Big(1+2\sum_{\mu=1}^{m}w_{0\mu}\Big)} \tag{33}$$

and to the low-pass transfer function

$$H(z) = H\cdot\frac{1+z^{-1}}{2}\prod_{\mu=1}^{m}\frac{1+2w_0z^{-1}+z^{-2}}{1+b_\mu^2z^{-2}}\ , \qquad H = \sqrt{\prod_{\mu=1}^{m}\frac{b_\mu^2}{2}Q} \tag{34}$$

and β_μ, b_μ are the limits ($a=0$, $\alpha_0\to\infty$) of the formulae from Table I

$$\alpha_\mu = 0\ , \quad \beta_\mu = \sqrt{\frac{w_{0\mu}^2-k'^2}{1-w_{0\mu}^2}}\ , \quad a_\mu = 0\ , \quad b_\mu = \frac{\sqrt{1-w_{0\mu}^2}}{k+\sqrt{w_{0\mu}^2-k'^2}}\ . \tag{35}$$

3. Complementary filter pairs and all-pass decomposition

Consider the low-pass transfer function of an odd order equiripple IIR filter

$$H(z) = H_0 \frac{1+z^{-1}}{1-a_0 z^{-1}} \prod_{\mu=1}^{m} \frac{z^{-2}+2w_{0\mu}z^{-1}+1}{(a_\mu^2+b_\mu^2)z^{-2}-2a_\mu z^{-1}+1} \tag{36}$$

which is to be decomposed as

$$H(z) = \frac{1}{2}[A_1(z)+A_2(z)] \ . \tag{37}$$

The stable poles (24) are assumed to be contained in (36) and (37). Let $A_1(z)$ and $A_2(z)$ be yet unknown all-pass transfer functions described as

$$A_1(z) = \frac{z^{-(m+1)}a_1(z^{-1})}{a_1(z)}, \quad A_2(z) = \frac{z^{-m}a_2(z^{-1})}{a_2(z)} \ . \tag{38}$$

In this section we show how the all-pass functions are determined from a designed low-pass transfer function and from the distribution of its poles. Since H (z) has zeros on the unit circle only and

$$H(z^{-1}) = \frac{1}{2}\Big[A_1(z^{-1})+A_2(z^{-1})\Big] = \frac{1}{2}\frac{A_1(z)+A_2(z)}{A_1(z)A_2(z)} \tag{39}$$

we readily get

$$A_1(z)A_2(z) = \frac{H(z)}{H(z^{-1})} = \frac{1}{z^{2m+1}} \cdot \frac{1-a_0 z}{1-a_0 z^{-1}} \prod_{\mu=1}^{m} \frac{(a_\mu^2+b_\mu^2)-2a_\mu z+1}{(a_\mu^2+b_\mu^2)z^{-2}-2a_\mu z^{-1}+1} \ . \tag{40}$$

Equation (40) confirms that the poles of the product of the all-pass transfer functions are identical with the poles of the transfer function. The low-pass magnitude

$$Q(w) = H(z)H(z^{-1}) = \frac{1}{2} + \frac{1}{4}\Big[A_1(z)A_2(z^{-1})+A_1(z^{-1})A_2(z)\Big] \tag{41}$$

can be written in the alternative form

$$Q(w) = \frac{1}{2} + \frac{1}{2} \cdot \frac{1-\epsilon^2F^2(w)}{1+\epsilon^2F^2(w)} = \frac{1+R(w)}{2} \tag{42}$$

which defines the reactance-like function

$$R(w) = \frac{1}{2}\Big[\frac{A_1(z)}{A_2(z)} + \frac{A_2(z)}{A_1(z)}\Big] \tag{43}$$

for

$$w = \frac{1}{2}(z+\frac{1}{z}) \ . \tag{44}$$

By inverting (43) we obtain

$$\frac{A_1(z)}{A_2(z)} = R(w) + \sqrt{R^2(w)-1}\Big|_{w=\frac{1}{2}(z+\frac{1}{z})} = \frac{1-\mathrm{j}\epsilon F(w)}{1+\mathrm{j}\epsilon F(w)}\Big|_{w=\frac{1}{2}(z+\frac{1}{z})} \ . \tag{45}$$

In order to satisfy equation (45) the distribution of the poles $a_\mu \pm \mathrm{j}\ b_\mu$ between $A_1(z)$ and $A_2(z)$ is derived from the factorized characteristic equation (12)

$$\Big[1 + \mathrm{j}\epsilon \cdot sn(n\frac{\mathbf{K}(k_1)}{\mathbf{K}(k)}u \mid k)\Big]\Big[1 - \mathrm{j}\epsilon \cdot sn(n\frac{\mathbf{K}(k_1)}{\mathbf{K}(k)}u \mid k)\Big] = 0\ . \tag{46}$$

The solutions of (12) are decoupled into the following sets. The first factor of (46) provides the equation

$$1 + \mathrm{j}\epsilon \cdot sn(n\frac{\mathbf{K}(k_1)}{\mathbf{K}(k)}u \mid k_1) = 0 \tag{47}$$

which contains the stable poles belonging to $A_1(z)$ (cf. (45))

$$w_{02\nu} = dn\Big(\mathrm{j}v_\infty + \frac{4\nu}{n}\mathbf{K}(k) \mid k\Big) \tag{48}$$

while the other zeros of (47)

$$k' dn\Big(\mathrm{j}v_\infty + \frac{4\nu+1}{n}\mathbf{K}(k) \mid k\Big)$$

for $\nu = 0, 1, \ldots, [m/2]$ produce poles which fall outside the unit circle. Similarly, the second factor provides the equation

$$1 - \mathrm{j}\epsilon \cdot sn\Big(n\frac{\mathbf{K}(k_1)}{\mathbf{K}(k)}u \mid k_1\Big) = 0 \tag{49}$$

which belongs to the all-pass $A_2(z)$ (cf. (45)). Hence, we get the stable poles

$$w_{02\nu+1} = dn\Big(\mathrm{j}v_\infty + \frac{4\nu+2}{n}\mathbf{K}(k) \mid k\Big) \tag{50}$$

while

$$k' dn\Big(\mathrm{j}v_\infty + \frac{4\nu+3}{n}\mathbf{K}(k) \mid k\Big)$$

for $\nu = 0, 1, \ldots, [m/2]$ are those solutions occuring outside the unit circle. Considering only the first interval of periodicity $[0, 2\mathbf{K}(k)]$ of the elliptic function $dn(u|\,k)$, the factorized characteristic equation (46) provides stable poles which can be denoted as even ($\mu = 2\nu$) and odd ($\mu = 2\nu + 1$) poles, respectively. Comparing with (13),(14) and (15) we obtain the same set of poles

$$\alpha_\mu \pm \mathrm{j}\beta_\mu = dn\Big(\pm \mathrm{j}v_\infty + \frac{2\mu}{n}\mathbf{K}(k) \mid k\Big) \tag{51}$$

($\mu = 0, 1, \ldots, m$) for which the addition theorem [1] of the function $dn(x + y \mid k)$ should be used. It consequently means that the poles of the all-pass functions $A_1(z)$ and $A_2(z)$ alternate, e.g.

$$A_1(z) = \frac{z^{-1} - a_0}{1 - a_0 z^{-1}} \prod_{\nu=1}^{[m/2]} \frac{z^{-2} - 2a_{2\nu}z^{-1} + (a_{2\nu}^2 + b_{2\nu}^2)}{(a_{2\nu}^2 + b_{2\nu}^2)z^{-2} - 2a_{2\nu}z^{-1} + 1} \tag{52}$$

$$A_2(z) = \prod_{\nu=1}^{[m/2]} \frac{z^{-2} - 2a_{2\nu+1}z^{-1} + (a_{2\nu+1}^2 + b_{2\nu+1}^2)}{(a_{2\nu+1}^2 + b_{2\nu+1}^2)z^{-2} - 2a_{2\nu+1}z^{-1} + 1}\ . \tag{53}$$

The bireciprocal case formulae are easily deduced letting $a_0 = a_{2\nu} = a_{2\nu+1} = 0$. The function $R(w)$ alone determines the low-pass and high-pass magnitudes since from (41) we have

$$Q_{L,H}(w) = \frac{1}{2} \pm \frac{1}{4}\Big[A_1(z)A_2(z^{-1}) + A_1(z^{-1})A_2(z)\Big] = \frac{1 \pm R(w)}{2} \quad . \tag{54}$$

The function R(w) has several important properties :

- The poles are identical to those of the transfer function.
- The real zero determines the 3dB cross-section point

$$w_{dB} = \frac{1}{k' dc(v_{dB} \mid k')} \tag{55}$$

  where

$$v_{dB} = \frac{\mathbf{K}(k)}{n\mathbf{K}(k_1)} \pm \Big(\arcsin \frac{\sqrt{1-\epsilon^2}}{k'_1} \mid k'_1 \Big) \quad . \tag{56}$$

- For the bireciprocal case, it has alternating purely imaginary poles and zeros and, therefore, in the w-domain it resembles the reactance function

$$R(w) = \prod_{\mu=1}^{m} \frac{sn^2\Big(\frac{2\mu-1}{n}\mathbf{K}(k) \mid k\Big)}{sn^2\Big(\frac{2\mu}{n}\mathbf{K}(k) \mid k\Big)} \; w \; \prod_{\mu=1}^{m} \frac{w^2 + cs^2\Big(\frac{2\mu-1}{n}\mathbf{K}(k) \mid k\Big)}{w^2 + cs^2\Big(\frac{2\mu}{n}\mathbf{K}(k) \mid k\Big)} \quad . \tag{57}$$

4. Lattice Wave Digital Filter Design Cook-book

The equations given in the text can be used for the straightforward approach to a two-fold set of the equiripple complementary filter pairs. Correspondingly, the cookbook is divided into two sections. The first section deals with the standard equiripple approximation (unconstrained) to a complementary pair. The second section contains design formulae for the constrained approximation to a complementary pair. The former case was thoroughly studied and reported by L. Gazsi [4] while the latter was developed recently [10] yet not applied to the lattice wave filter design. These two design procedures overlay each other and they are separated for the sake of cookbook to make the reading comprehensible (Fig. 2).

4.1. Unconstrained standard design

1. Given the passband limit ω_p, the stopband limit ω_s and the sampling frequency ω_0, we find $\overline{\omega}_p$, $\overline{\omega}_s$ according to (4) and we get the constants a, b using (2),(3). Given the maximum attenuation in the passband a_p and the minimum attenuation in the stopband a_s, the discrimination factor k_1 from (5) and the passband ripple ϵ from (6) are obtained.
2. If $a \neq 0$ then the low-pass to low-pass transformation $\overline{w} \rightarrow w$ is done and k' from (7) determined.

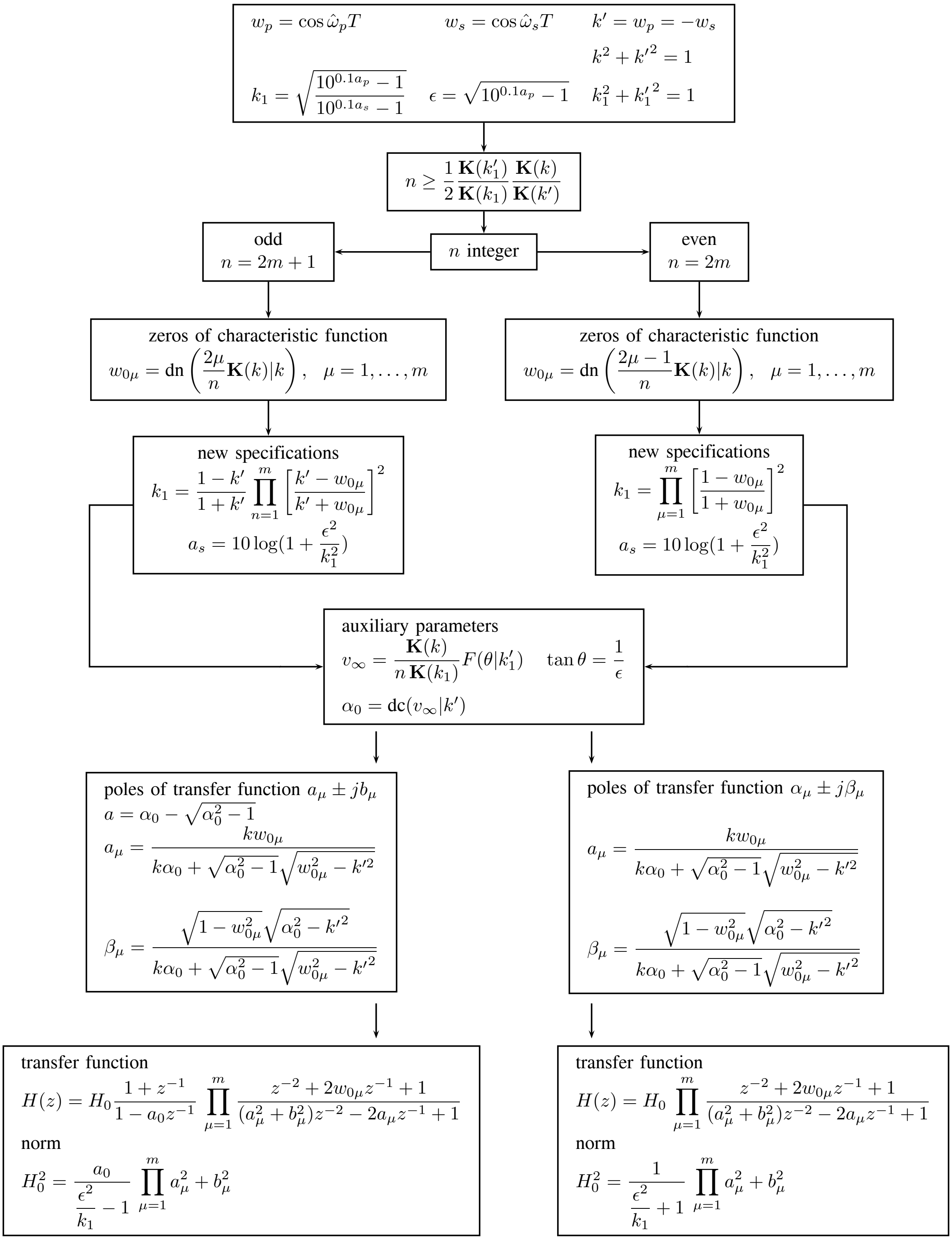

Figure 2: Summary of the design steps in the z-domain.

3. Using (9) the odd integer $n = 2m + 1$ is estimated.
4. The zeros of the characteristic function $w_{0\mu}$ from (10) are computed and the new discrimination factor k_1 is recomputed from (28).
5. The poles of the transfer magnitude $\alpha_0, \alpha_\mu, \beta_\mu$ are evaluated from (13), (14) and (15).
6. The poles of the transfer function a_0, a_μ, a_μ are computed from (26), (24) and the all-pass sections (52), (53) are formed.
7. If $a = 0$ then the multiplier coefficientes read

$$\gamma_0 = a_0 = \alpha_0 - \sqrt{\alpha_0^2 - \gamma} \tag{58}$$

$$\gamma_{2\mu-1} = -(a_\mu^2 + b_\mu^2) = -\frac{k\alpha_0 - \sqrt{\alpha_0^2 - 1}\sqrt{\alpha_{0\mu}^2 - k'^2}}{k\alpha_0 + \sqrt{\alpha_0^2 - 1}\sqrt{\alpha_{0\mu}^2 - k'^2}} \tag{59}$$

$$\gamma_{2\mu} = \frac{2a_\mu}{1 + a_\mu^2 + b_\mu^2} = \frac{w_{0\mu}}{\alpha_0} \tag{60}$$

8. If $a \neq 0$ then the multiplier coefficients are expresed in the original z-domain as

$$\overline{\gamma}_0 = \overline{a}_0, \quad \overline{\gamma}_{2\mu-1} = -(\overline{a}_\mu^2 + \overline{b}_\mu^2), \quad \overline{\gamma}_{2\mu} = \frac{2\overline{a}_\mu}{1 + \overline{a}_\mu^2 + \overline{b}_\mu^2} \tag{61}$$

where

$$\overline{a}_0 = \frac{a_0 + b}{1 + ba_0} \tag{62}$$

and $\overline{a}_\mu$,$\overline{b}_\mu$ are

$$\overline{a}_\mu = \frac{b + a_\mu(1 + b^2) + b(a_\mu^2 + b_\mu^2)}{(1 + b\,a_\mu)^2 + b^2 b_\mu^2}\,, \quad \overline{b}_\mu = b_\mu \frac{1 - b^2}{(1 + b\,a_\mu)^2 + b^2 b_\mu^2}\;. \tag{63}$$

4.2. Constrained design

The design starts with the steps one to four of the former procedure. Then we perform

1. The constraint (27) is imposed giving the available passband ripple ϵ and consequently the new atttenuation levels a_p, a_s from (29).
2. The poles of the transfer magnitude $\alpha_0, \alpha_\mu, \alpha_\mu$ are evaluted from (30), (14) and (15), respectively.
3. The poles of the transfer function $a_0 = -b, a_\mu, b_\mu$ are computed from a_μ, b_μ.

4. The condition $a = 0$ leads to the bireciprocal case with the multiplier coefficients

$$\gamma_0 = 0, \quad \gamma_{2\mu-1} = 0, \quad \gamma_{2\mu} - b_\mu^2 = -\frac{1 - w_{0\mu}^2}{\left[k + \sqrt{w_{0\mu}^2 - k'^2}\right]^2} \quad . \tag{64}$$

5. If $a \neq 0$ the re-transformation back to the original $\overline{z}$-domain occurs for $\overline{a}_0 = 0, \overline{a}_\mu, \overline{b}_\mu$ and the corresponding nonzero multiplier coefficientes are given by (61).

5. Design Examples

The design formulae are demonstrated by some particular examples. We use the same adaptors as used by L. Gaszi [4] for the all-pass sections (Section 4). Corresponding signal flow charts are depicted in Fig. 3. The all-pass structure of the wave digital filter with an odd degree equiripple transfer function is shown in Fig. 4. Figure 5 contains the all-pass decomposition for the bireciprocal case of the allied specifications.

5.1. Example 1 (unconstrained direct design)

We wish to design the all-pass structure of the equiriple wave digital filter specified by $a_p = 0.447dB$, $a_s = 24dB$, $\overline{w}_p = -0.44$ and $\overline{w}_s = -0.5$.

The integer solution of (9) $n = 5$ provides $a_p = 0.447$[dB] and $a_s = 24.49$[dB]. The multiplier coefficients $\overline{\gamma}$ and α are

$$\begin{aligned}
\gamma_0 &= -0.000116, & \alpha_1 &= -0.000116,\\
\gamma_1 &= -0.595288, & \alpha_1 &= 0.404712,\\
\gamma_2 &= -0.350361, & \alpha_1 &= -0.350361,\\
\gamma_3 &= -0.937179, & \alpha_1 &= 0.062821,\\
\gamma_4 &= -0.449656, & \alpha_1 &= -0.449656\,.
\end{aligned}$$

The magnitude responses of the designed wave digital filter is depicted in Fig. 6. The filter all-pass structure is shown in Fig. 7.

5.2. Example 2 (constrained design)

Design the equiriple wave digital filter and its all decomposition specified by $a_p = 0.447$[dB], $a_s = 24$[dB], $\overline{w}_p = -0.44$ and $\overline{w}_s = -0.5$ with the constraint $\alpha_0 = -1/a$.

We again obtain $n = 5$ and $a_p = 0.447$[dB] and $a_s = 24.49$[dB]. The multiplier coefficients $\overline{\gamma}$ and α are

$$\begin{aligned}
\gamma_0 &= 0.000000, & & \\
\gamma_0 &= -0.595314, & \alpha_3 &= 0.404686,\\
\gamma_0 &= -0.350312, & \alpha_4 &= -0.350312,\\
\gamma_0 &= -0.937185, & \alpha_5 &= 0.062815,\\
\gamma_0 &= -0.449647, & \alpha_6 &= -0.449647\,.
\end{aligned}$$

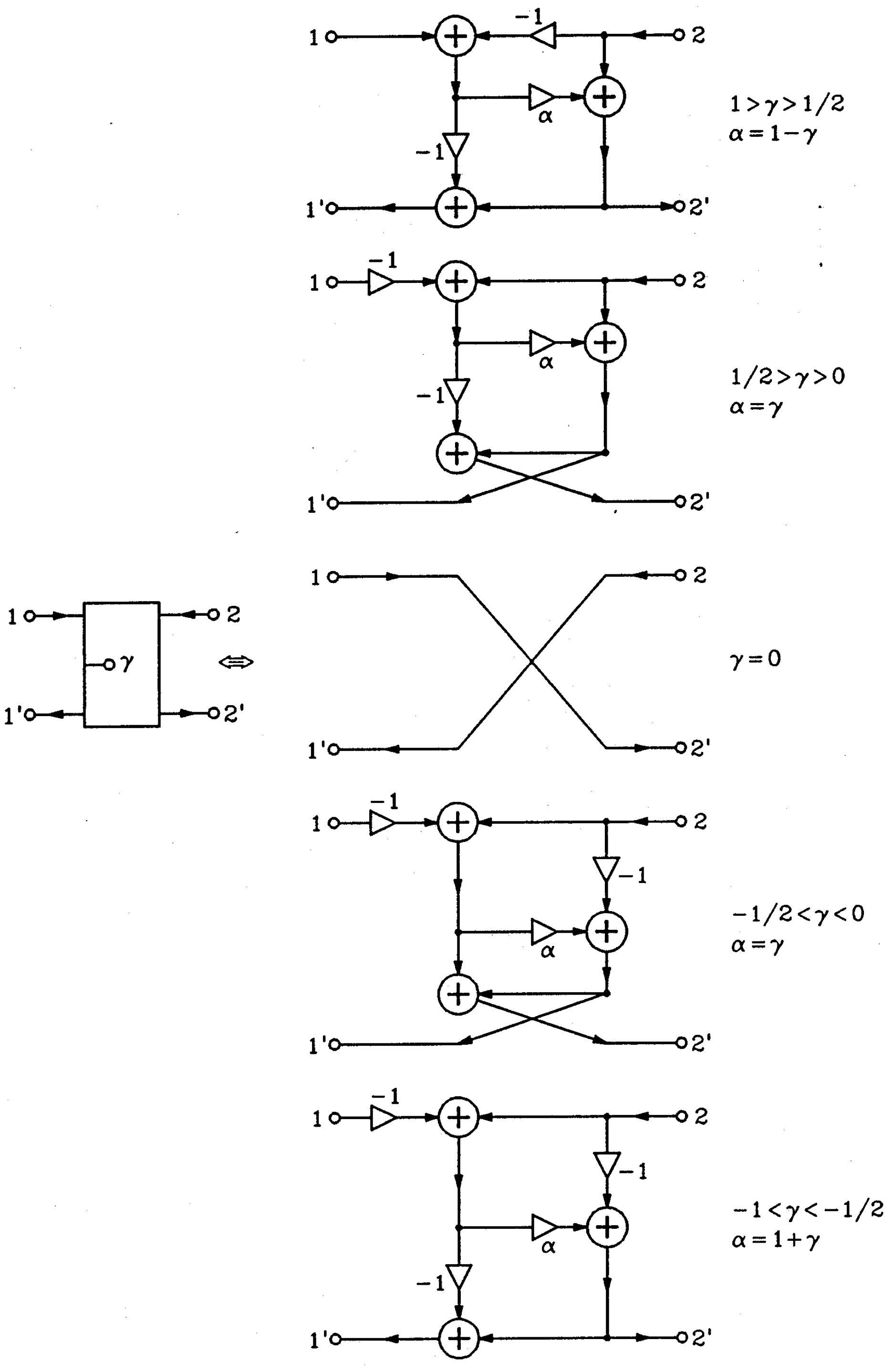

Figure 3: Signal flow diagram of the two-port adaptor.

5.3. Example 3 (bireciprocal case)

The required bireciprocal filter is specified by the two parameters $a_s = 0.017324$[dB] and $w_p = 0.44$ only. The constraint (27) yields for $n = 5$ the available attenuation levels

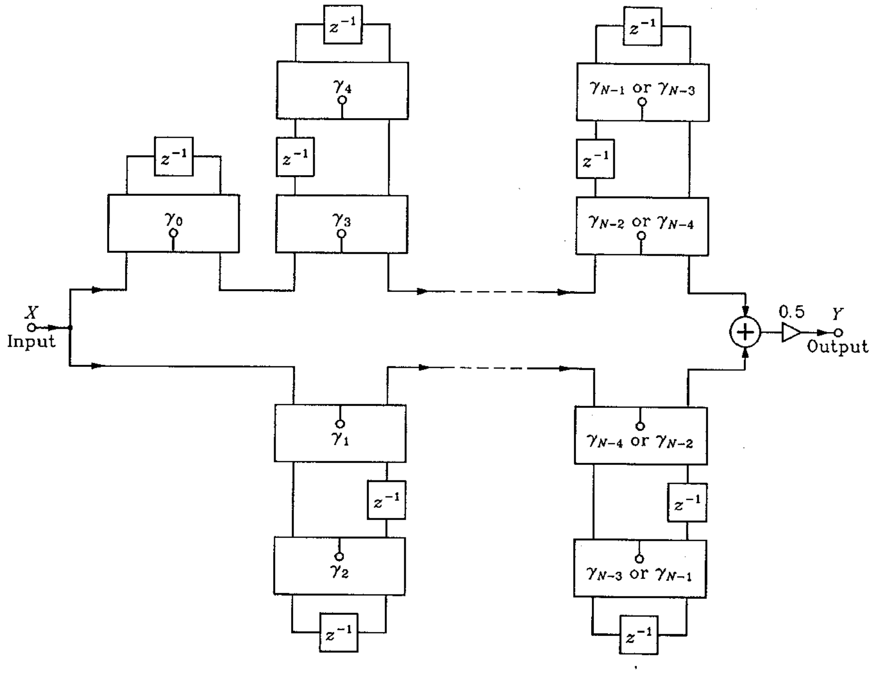

Figure 4: Block diagram of the lattice wave digital filter with cascaded all-pass sections, odd n.

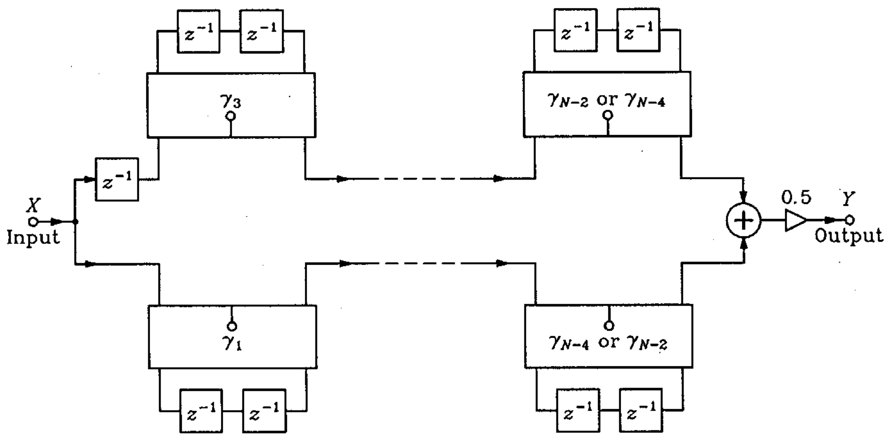

Figure 5: Block diagram of the bireciprocal lattice wave digital filter with cascaded all-pass sections for odd n.

$a_p = 0.0001846$[dB], $a_s = 43.72$[dB]. The corresponding all-pass structure is drawn in Fig. 8 for the following set of coefficientes γ and β

$$\begin{aligned} \gamma_0 &= 0.000000 \\ \gamma_1 &= -0.191227 \qquad \alpha_2 = -0.191227 \\ \gamma_2 &= 0.000000 \\ \gamma_3 &= -0.661072 \qquad \alpha_1 = 0.338927 \\ \gamma_4 &= 0.000000\ . \end{aligned}$$

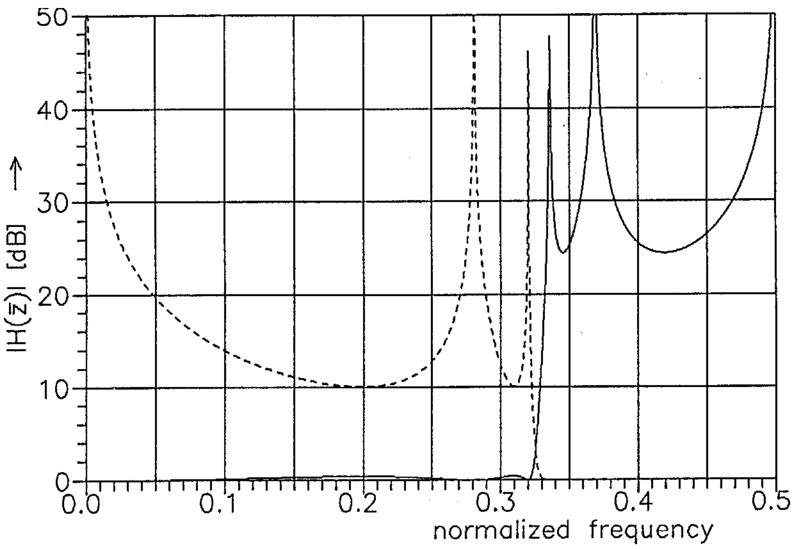

Figure 6: Attenuation responses in the $\overline{z}$-domain for the low-pass (solid line) and complementary high-pass (dashed line) filter.

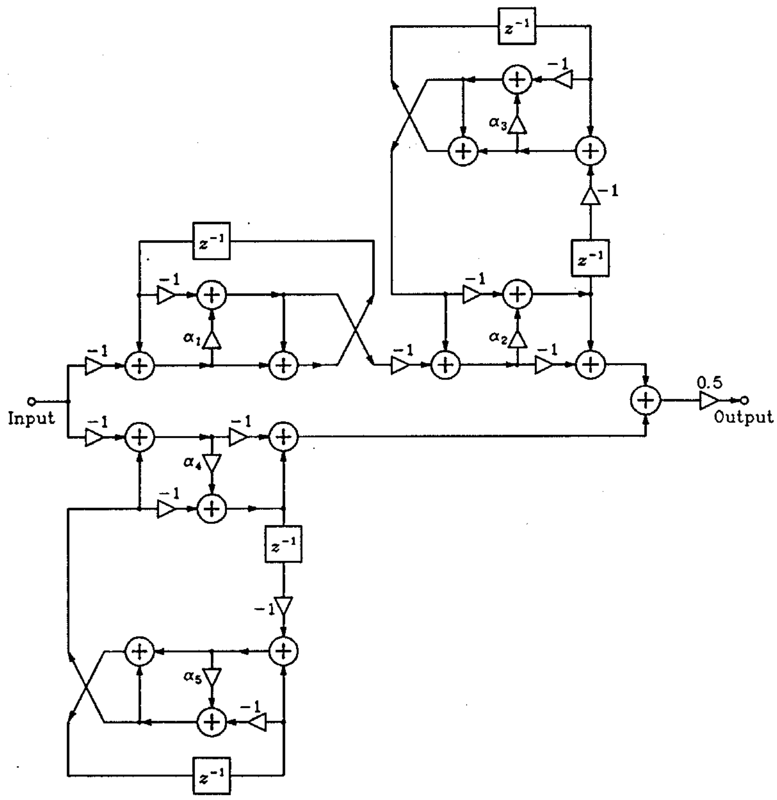

Figure 7: Signal flow of the low-pass filter.

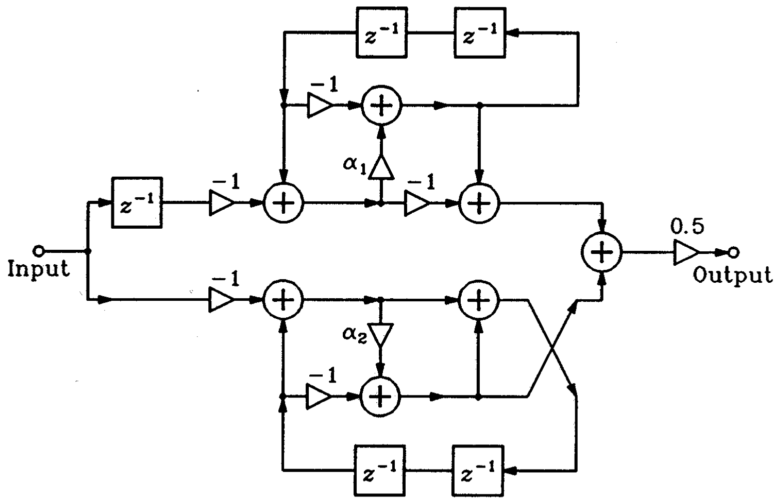

Figure 8: Signal flow diagram of the low-pass filter for bireciprocal case.

References

[1] M. Abramowitz, I. Stegun, *Handbook of mathematical functions,* Dover Publications, Newyork, 1972 Chapter 16 and 17.

[2] R.ansari and B. Liu, A class of low-noise computationally efficient recursive digital filters with applications to sampling rate alternations. *IEEE Trans. on Acustic, Speech, and Signal Processing*, vol. ASSP-33, No. 1, February 1985, pp. 90-97.

[3] A. Fettweis, Digital Filters: Theory and Practice. *Proc. IEEE*, vol. 74, No. 2, February 1986, pp. 270-327.

[4] L. Gazsi, Explicit Formulas for Lattice Wave Filter Structure. *IEEE Trans. on Circuits and Systems*, vol. CAS-32, No. 1, January 1985, pp. 68-88.

[5] A.H. Gray Jr., J.D. Markel, A normalized Digital filter Structure. *IEEE Trans. on Acustic, Speech, and Signal Processing*, vol. ASSP-23, No. 3, June 1977, pp. 268-277.

[6] S.K. Mitra, K. Hirano, Digital All-Pass Networks. *IEEE Trans. on Circuits and Systems*, vol. CAS-21, No. 5, September 1974, pp. 688-700.

[7] P. A. Regalia, S.K. Mitra, P.P. Vaidyanathan, The Digital All-Pass Filter:A versatile Signal Processing Building Block. *Proc. of IEEE*, vol. 76, No. 1, January 1988, pp. 19-37.

[8] R. Unbehauen, *Synthese elektrischer Netzwerke und Filter.* R. Oldenbourg Verlag, München, 1988, Part V, Chapter 4.

[9] P.P. Vaidyanathan, S.K. Mitra, K. Neuvo, A new approach to the realization of low sensitivity IIR Digital Filters. *IEEE Trans. on Acustic, Speech, and Signal Processing*, vol. ASSP-34, No. 2, April 1986, pp. 350-361.

[10] M. Vlček, R. Unbehauen, Analytical Solution for Design of IIR Equirriple Filters. *IEEE Trans. on Acustic, Speech, and Signal Processing*, vol. ASSP-37, No. 10, October 1989, pp. 1518-1531.

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Study visit -Institut National of Telecommunications (INT), 1996, Evry, France - 1 month
Study visit -University of Kielce, Poland, 2001 - 1 month
Study visit -University of Santa Barbara, California, 2005 - 1 week, Prof. S.K. Mitra
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Lectures: Digital Signal Processing, Data Transmission
Supervising of graduate students : over 20 succesfully graduated students
Supervising of PhD. students : 7 successful students (2 foreign students), currently 5 students

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J. Prchal, B. Šimák, *Digital Signal Processing.* Praha, CTU 2000, 224 pages, ISBN 80-01-02149-1.
B. Šimák, J. Vodrážka, J. Svoboda, *Digital Subscriber Line – xDS - 1.* Praha, Sdělovací technika, 2005, 142 pages, ISBN 80-86645-07-X.
Vodrážka, J. - Šimák, B.: *Digital Subscriber Line – xDSL- 2.* Praha: Sdělovací technika, 2008. 156 s. ISBN 80-86645-16-9.

Publications in the last 5 years
Zahradník, P. - Šimák, B. - Vlček, M.: Fast Design of 2-D Extremely Narrow Bandstop FIR Filters. In *MWSCAS 2004* [CD-ROM]. Piscataway: IEEE, 2004, s. II-69-II-72. ISBN 0-7803-8347-8.
Vlček, M. - Šimák, B.: Design of IIR Equiripple Filters in z-Domain. In *Abstracts of Papers EUROCON 2005*. Belgrade: School of Electrical Engineering, 2005, p. 14. ISBN 86-7466-218-8.
Vlček, M. - Šimák, B.: Design of IIR Equiripple Filters in z-Domain. In *EUROCON 2005 - Proceedings* [CD-ROM]. Piscataway: IEEE, 2005, p. SS_48. ISBN 1-4244-0050-3.
Vodrážka, J. - Šimák, B.: Theoretical Limits of DSL Lines. In *Communication and Information Technologies - Conference Proceedings*. Liptovský Mikuláš: Akadémia ozbrojených síl, 2005, p. 243-246. ISBN 80-8040-269-8.
Zahradník, P. - Vlček, M. - Šimák, B.: Analytical Design of Maximally Flat Notch FIR Filters for Communication Purposes. In Networking - ICN 2005. Berlin: Springer-Verlag, 2005, díl 2, s. 392-400. ISBN 3-540-25338-6.
B. Šimák, M. Vlček, P. Zahradnik, "Analytical Design of Bandstop FIR Filters for Image Processing". *2006 2nd International Symposium on Communicati-ons, Control and Signal Processing.* March 2006, Marrakech, Morocco. Anaheim: ACTA Press, 2006, p. 465-468. ISBN 0-88986-585-X.
P. Zahradnik, B. Šimák, M. Vlček, "Fast Design of Precise Equiripple Notch FIR Filters". 5th *International Conference on Networking ICN'06.* April 2006, Mauritius. In print.
Šimák, B. - Vlček, M. - Zahradník, P.: Analytical Design of Bandstop FIR Filters for Image Processing. In *ISCCSP 2006 Proceedings* [CD-ROM]. Monterey: IEEE Circuits and Systems Society, 2006, p. cr1182. ISBN 2-908849-17-8.
Šimák, B.: Improvements of the Subject Digital Signal Processing in Telecommunication. In *Research in Telecommunication Technology 2006 - Proceedings* [CD-ROM]. Brno: Vysoké učení technické v Brně, 2006, vol. I, p. 174-175. ISBN 80-214-3243-8.

Zahradník, P. - Šimák, B. - Vlček, M.: Design of Maximally Flat Notch FIR Filters. In *IWSSIP 2006 and COST 292*. Košice: HARLEQUIN s.r.o., 2006, p. 357-360. ISBN 80-89082-09-2.
Zahradník, P. - Šimák, B. - Vlček, M.: Fast Design of Precise Equiripple Notch FIR Filters. In *Proceedings ICN 2006 / ICONS 2006 / MCL 2006* [CD-ROM]. Piscataway: IEEE, 2006, p. 157. ISBN 0-7695-2570-9.
Zahradník, P. - Šimák, B. - Vlček, M.: Nearly Equiripple Half-Band FIR Filters. In *Proceedings of the Eighth IASTED International Conference on Signal and Image Processing* [CD-ROM]. Anaheim: ACTA Press, 2006, p. 465-468. ISBN 0-88986-585-X.
Primechaev, S. - Frolov, A. - Šimák, B.: Scene Change Detection Using DCT Features in Transform Domain Video Indexing. In *IWSSIP 2007 & EC-SIPMCS 2007* [CD-ROM]. Maribor: University of Maribor, 2007, p. S13-02. ISBN 978-961-248-029-5.
Primechaev, S. - Frolov, A. - Šimák, B.: Scene Change Detection Using DCT Features in Transform Domain Video Indexing. In *2007 IWSSIP & EC-SIPMCS (Abstracts)*. Maribor: University of Maribor, 2007, p. 47. ISBN 978-961-248-030-1.
Sakhnov, K. - Šimák, B.: Performance Evaluation of LMS based Algorithms used in Echo Cancellation. In *RTT 2008* [CD-ROM]. Bratislava: STU v Bratislave, 2008, ISBN 978-80-227-2939-0.
Zahradník, P. - Vlček, M. - Šimák, B.: DC-Notch FIR Filters for Zero-IF Receivers. In *Sixth International Conference on Networking (ICN 2007)* [CD-ROM]. Los Alamitos: IEEE Computer Society Press, 2007, p. 069. ISBN 0-7695-2805-8.
Šimák, B.: Starter Kits for Digital Signal and Image Processing in Communication. In *RTT 2008* [CD-ROM]. Bratislava: STU v Bratislave, 2008, ISBN 978-80-227-2939-0.
Zahradník, P. - Šimák, B. - Vlček, M.: Simple Daughter Cards for TI DSP Starter Kits for the Education in Image and Video Processing. In *European DSP Education and Research Symposium - Proceedings and Literature* [CD-ROM]. Houston: Texas Instruments, 2008, ISBN 978-0-9552047-2-2.
Zahradník, P. - Vlček, M. - Šimák, B.: Extended Analytical Design of Equiripple Comb FIR Filters. In *Proceedings of the 3rd International Symposium on Communications, Control and Signal Processing* [CD-ROM]. New York: Institute of Electrical and Electronics Engineers, 2008, p. 282-287. ISBN 978-1-4244-1688-2.

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