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Intelligent Transport Systems - Design Methodology and Effectiveness Assessment

Inteligentní dopravní systémy - metody návrhu a hodnocení účinnosti

Summary:

The processes in the **ITS (Intelligent Transport Systems)** architecture are defined by chaining system components through the information links. The chains of functions (processes) are mapped in physical subsystems or modules and information flows between functions specify the communication links between subsystems or modules. The functions' grouping taking into account the market availability of modules/applications yields into definition of ITS market packages.

If time, performance or other constrains are assigned to different functions and information links, the result of the analysis is represented by table of different, sometimes even contradictory system requirements assigned to each physical subsystem (module) and physical communication links between subsystems.

Referring to ITS architecture and ITS market packages the mathematical tool of modelling ITS systems and subsystems is introduced. Mathematical tool covers the estimation of performance parameters, dynamical model identification, fuzzy-linguistic approximation, classification and modelling of large-scale systems by complex-multi-models, methods of data reduction, fusion and comparison.

With help of mathematical tools the appropriate telecommunication environment can be statically/dynamically selected or switched, data can be pre-processed and reduced in on-board unit, etc. and the ITS technical design can be optimized.

The ITS designer must also take into account the economical aspects. Naturally, ITS effectiveness definition is an essential issue therefore there is a strong focus placed on it. On that account internationally reputable methodology of cost-benefit evaluation (CBA) is chosen and connected with the effectiveness definition as well, so the effectiveness values are represented by e.g. Net present value, Internal rate of return, Pay-off period, etc.

On the negative impact side (cost side) is this procedure quite simple. It is given by knowledge of investment and operating costs of almost all ITS applications and by existing of no obstructions involved in their enumeration. On the benefit side is the situation much more complicated because benefits have to be expressed in their natural units first and ex-post transformed into a monetary form in the second step.

Accommodation of both these evaluation views is possible to achieve using so called fuzzy-linguistic approximation, which is an approach proposed for the calculating of fuzzy values coming from the fuzzy variables defined – in our case from the qualitative and socio-economic indicators. It is presumed that evaluation procedure is done separately for cost and benefits. Separate models for costs and benefits originate that way and represent basic input into the final effectiveness calculation (defined by CBA indicators).

Souhrn:

Procesy v rámci architektury inteligentních dopravních systémů (ITS) jsou definovány jako řetězení systémových prvků prostřednictvím informačních vazeb. Řetězce funkcí (procesů) jsou mapovány na fyzické subsystémy a moduly a informační toky mezi nimi definují komunikační prostředí mezi fyzickými subsystémy nebo moduly. Shlukováním dílčích funkcí dle dostupnosti produktů a služeb na trhu vede na definici tzv. komerčních ITS balíčků.

K jednotlivým funkcím a procesům či informačním vazbám je nutno přiřadit systémové (performační) parametry. Výsledkem této analýzy je seznam často protichůdných požadavků na navrhovaný ITS systém. Systémové parametry jsou základním požadavkem pro návrh fyzických modulů a komunikačních vazeb.

Na základě vzniklé ITS architektury a komerčních balíčků je možno ITS systém začít modelovat. Popsané matematické metody umožňují odhadovat systémové (performační) parametry, modelovat dynamické vlastnosti systému včetně identifikace modelu systému, fuzzy-lingvistické aproximace v případě pouhých expertních znalostí, klasifikace či modelování rozsáhlých systémů pomocí komplexních pravděpodobnostních multi-modelů a v neposlední řadě i redukce, porovnávání či fůze dat.

Pomocí výše uvedených matematických nástrojů lze provádět statickou či dynamickou volbu telekomunikačního prostředí, předzpracovávat a redukovat data v palubních jednotkách, atd. a optimalizovat tak návrh celého ITS systému.

Při návrhu ITS systému je nutno též zohlednit ekonomické aspekty. Přirozeně je definice účinnosti ITS systému základním parametrem. S ohledem na vzájemnou mezinárodní porovnatelnost ITS systémů byla zvolena CBA (Cost-Benefit Analyze) jako základní metoda stanovení účinnosti ITS včetně svých částí NPV (Net Present Value), IRR (Internal Rate of Return), atd.

Na straně nákladů je metodika poměrně jednoduchá, neboť dochází ke sčítání investičních a provozních nákladů ITS aplikací, které lze odhadnout. Na straně přínosů je situace složitější, neboť přínosy musí být odhadovány ve svých jednotkách a teprve poté převedeny na finanční hodnoty.

Modelování přínosů a nákladů ITS systémů je možno provádět pomocí fuzzy-lingvistické aproximace se zahrnutím jak kvantitativních, tak i kvalitativních indikátorů. Metodika využívá expertní znalosti, které jsou zvlášť tvořeny pro nákladové a přínosové indikátory. Jejich modely jsou základem pro výpočet ITS účinnosti.

Keywords:

Intelligent transport systems, ITS, transport telematics, telematics, ITS architecture, ITS effectiveness, ITS design methodology, performance parameters, classification, data reduction,, multi-models, ITS assessment.

Klíčová slova:

Inteligentní dopravní systémy, ITS, dopravní telematika, ITS architektura, ITS účinnost, metodika návrhu ITS, systémové parametry, klasifikace, redukce dat, multi-modelování,zhodnocení ITS.

Content:

1. Introduction

Intelligent Transport Systems (ITS) are concerned with the use of new information, sensor and communication technologies to support transport services and applications across all modes. The development of ITS, in accordance with ERTICO and the European Commission view, provides an opportunity to apply advanced technology to systems and methods of transport for efficient, comfortable and safer highways, railways, inland waterways, airports, ports and linkages between these different types of transport.

Based on a wide vision of ITS deployment throughout Europe, the Trans-European Network for Transport (TEN-T) aims at establishing appropriate interconnection, interoperability and accessibility between services both on long-distance routes and in conurbation areas, providing an important step forward in implementation

The start period of ITS is characterized by strong investments for research and equipment in the road domain by private and public actors. It was in this period that the 3 biggest ITS organizations were created: ITS-America, ERTICO-ITS Europe and ITS-Japan.

Then came the experimental development period characterized by a major concern on interoperability of systems, which led public authorities to promote the definition of common architecture for ITS systems. In Europe it began with the EC (European Commission) programme “Advanced Road Transport Telematics” with Euro-regional cross-border projects in order to test inter-operability of road traffic management and information systems. In the railway domain, EC launched the ERTMS project (European Rail Traffic Management System) to improve effectiveness and inter-operability of signalling systems. In parallel, experimentations of e-ticketing and e-payment for public transport and Electronic Toll Collection (ETC) systems for toll motorway were conducted.

The following period of development for market and extension of ITS to all transport modes is characterized by two main preoccupations in Europe as in the USA or Japan: safety and sustainability.

The main problem of ITS deployment still remains the design methodology and effectiveness assessment of ITS projects. Chapter 2 presents the design methodology covering ITS architecture and performance evaluation together with example of ITS applications cluster. The result of ITS design are the specified ITS market packages being basic brick-boxes of ITS system. Chapter 3 introduces mathematical tools for the appropriate telecommunication environment selection, data pre-processing and reduction and for the ITS technical optimization. Chapter 4 presents assessment and evaluation of ITS effectiveness. Chapter 5 summarizes the conclusion and chapter 6 the references.

2. ITS design methodology

2.1 ITS architecture

The ITS architecture reflects several different views of the examined system and can be divided into:

- Reference architecture - defines the main terminators of ITS system (the reference architecture yields to definition of boundary between ITS system and environment of ITS system),
- Functional architecture - defines the structure and hierarchy of ITS functions (the functional architecture yields to the definition of functionality of whole ITS system),
- Information architecture - defines information links between functions and terminators (the goal of information architecture is to provide the cohesion between different functions),
- Physical architecture - defines the physical subsystems and modules (the physical architecture could be adopted according to the user requirements, e.g. legislative rules, organisation structure, etc.),
- Communication architecture - defines the telecommunication links between physical devices (correctly selected communication architecture optimises telecommunication tools),
- Organisation architecture - specifies competencies of single management levels (correctly selected organisation architecture optimises management and competencies at all management levels).

The instrument for creating ITS architecture is the process analysis shown on Fig.1. The processes are defined by chaining system components through the information links. The system component carries the implicit system function (F1, F2, F3, G1, G2, G3, etc.). The terminator (e.g. driver, consignee, emergency vehicle) is often the initiator and also the terminator of the selected process.

The chains of functions (processes) are mapped on physical subsystems or modules (first process is defined with help of functions F1, F2 and F3 on Fig.1, second process is defined by chaining the functions G1, G2 and G3) and the information flows between functions that specifies the communication links between subsystems or modules. If the time, performance, etc. constrains are assigned to different functions and information links, the result of the presented analysis is the table of different, often contradictory, system requirements assigned to each physical subsystem (module) and physical communication link between subsystems.

From the viewpoint of the construction of the selected subsystem it is possible to consider a single universal subsystem fulfilling the most exacting system parameters, the creation of several subsystem classes according to a set of system parameters, creation of a modular subsystem where the addition of another module entails the increase of system parameters, etc.

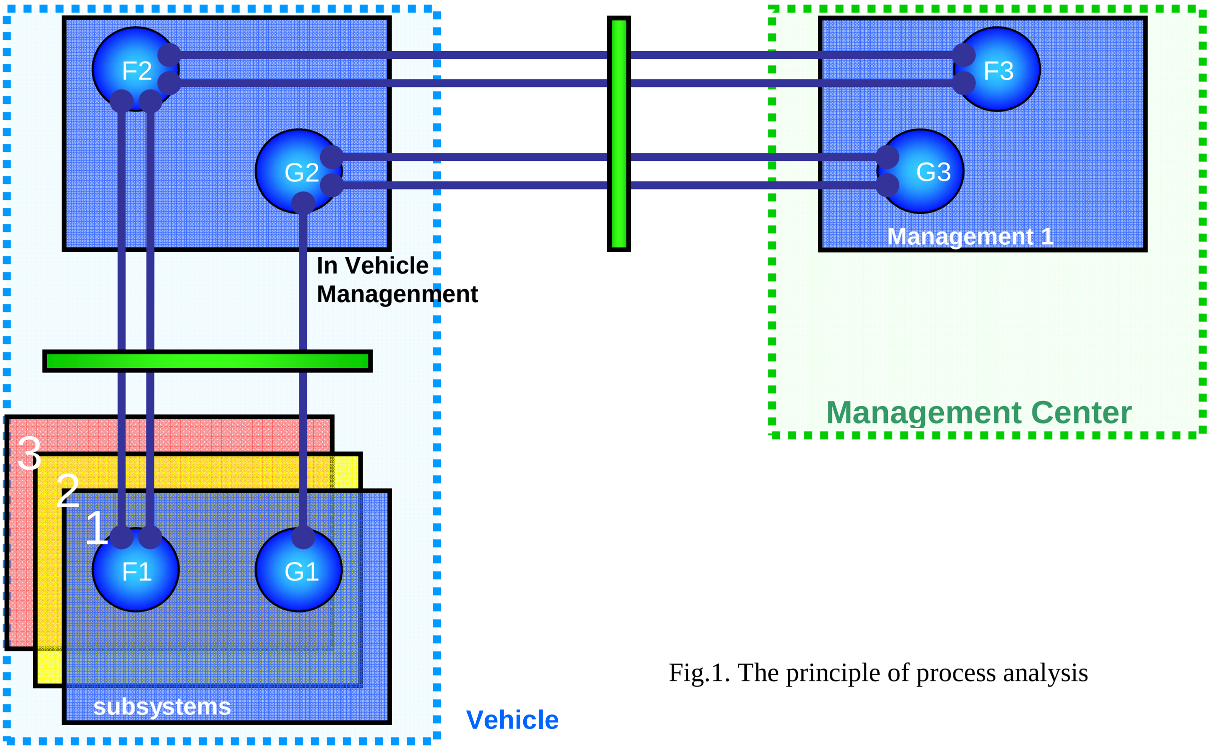

Fig.1. The principle of process analysis

The same principle may be applied while designing the telecommunication environment between selected subsystems (unified radio band frequency for all transport telematic applications, combination of individual transmission systems, combination of fixed and radio networks, etc.). In analogy with the subsystem design, the design of the telecommunication environment may be divided into several classes or, as the case may be, the transmission environment may be designed in a modular way when higher system parameters on the information transmission may be achieved by adding additional modules.

Similar situation applies to the other part of ITS system, or between ITS systems of different transport modes, e.g. road and railway transport. It is necessary to consider whether each transport mode has to have the selected subsystem alone available or whether there is an opportunity for sharing such subsystems, etc.

ITS architecture covers following makro-functions [18, 19]:

1. Provide Electronic Payment Facilities (toll collection system based on GNSS/CN, DSRC, etc.)

2. Provide Safety and Emergency Facilities (emergency call, navigation of rescue services, etc.)

3. Manage Traffic (traffic control, maintenance management, etc.)

4. Manage Public Transport Operations (active preferences of public transport, etc.)

5. Provide Advanced Driver Assistance Systems (car navigation services, etc.)

6. Provide Traveller Journey Assistance (personal navigation services, etc.)

7. Provide Support for Law Enforcement (speed limit monitoring, etc.)

8. Manage Freight and Fleet Operations (fleet management, monitoring of dangerous goods, etc.)

9. Provide Archive (location-based information, etc.)

The physical ITS architecture is shown on Fig.2.

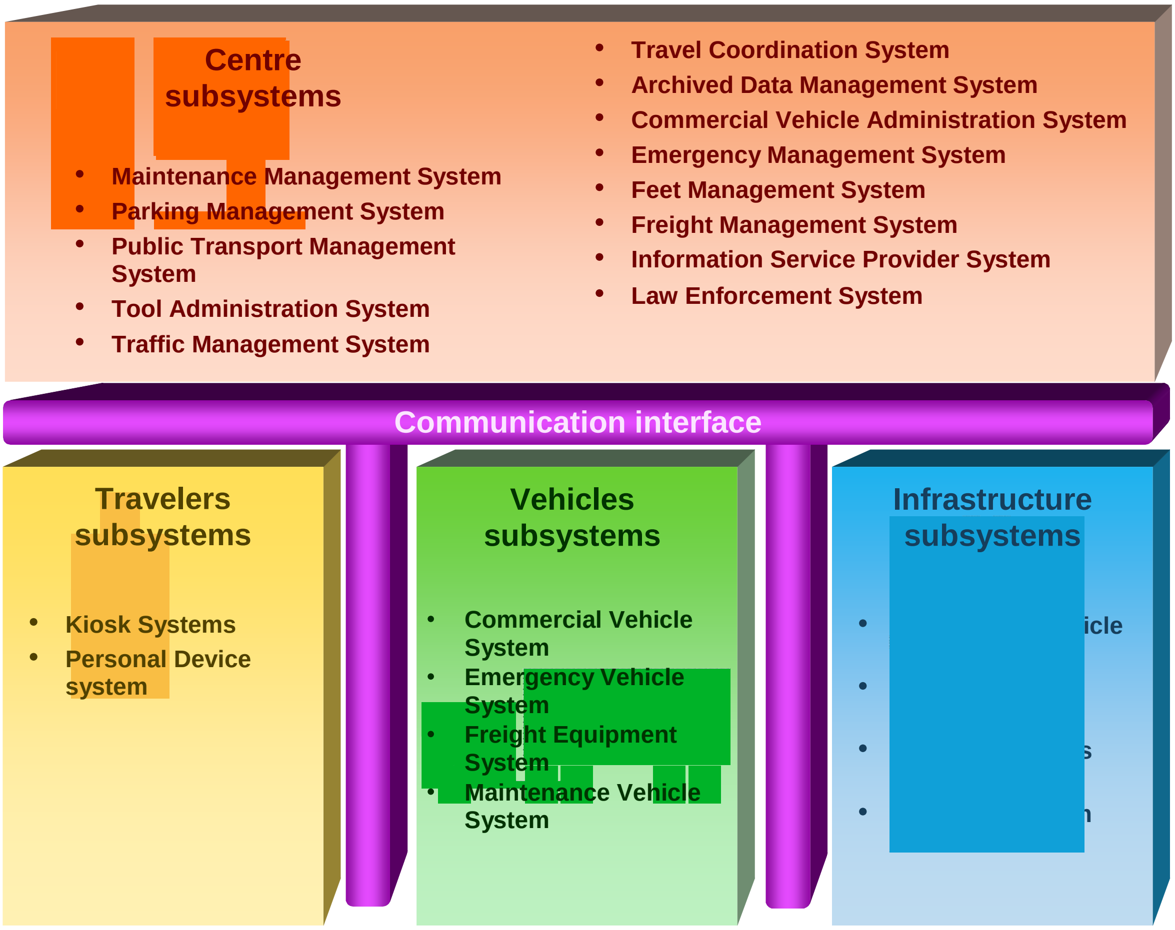

Fig. 2 ITS physical architecture

2.2 Performance parameters

First step in addressing the ITS architecture requirements should be the analysis and establishment of performance parameters in telematics applications, in co-operation with the end-users or with organisations like Railways Authority, Road and Motorways Directorates, etc.

The methodology for the definition and measurement of following individual system parameters is being developed in frame of the ITS architecture:

- Safety - risk analysis, risk classification, risk tolerability matrix, etc.
- Reliability - the ability to perform required function under given conditions for a given time interval.

- Availability - the ability to perform required function at the initialisation of the intended operation.
- Integrity - the ability to provide timely and valid alerts to the user when a system must not be used for the intended operation.
- Continuity - the ability to perform required function without non-scheduled interruption during the intended operation.
- Accuracy - the degree of conformance between a platform's true parameter and its estimated value, etc.

Substantial part of the system parameters analysis is represented by a decomposition of system parameters into individual sub-systems of the telematic chain. Part of the analysis is the establishment of requirements on individual functions and information linkage so that the whole telematic chain should comply with the above defined system parameters.

The completed decomposition of system parameters will enable the development of a methodology for a follow-up analysis of telematic chains according to the various criteria (optimisation of the information transfer between a mobile unit and processing centre, maximum use of the existing information and telecommunication infrastructure, etc.).

Mobility of the communication solution represents one of the crucial properties namely in context of frequently very specific demand on availability and security of the communication solution.

Data transmission capacity can represent due to possible high density of moving objects and limited wireless capacities critical system requirements, which can be resolved either by application of broadcasting regime of data distribution or by selective reduced data distribution with individual variable frequency where distance between objects represents simple but effective criteria for such data flow control.

Following communications performance parameters quantify telecommunication service quality:

- Availability – (i) Service Activation Time, (ii) Mean Time to Restore (MTTR), (iii) Mean Time Between Failure (MTBF) and (iv) VC availability,
- Delay - is an accumulative parameter effected by (i) interfaces rates, (ii) frame size, and (iii) load / congestion of all in line active nodes (switches).
- Packet/Frames Loss and
- Security.

Performance indicators described for communications applications must be transformed into telematic performance indicators structure, and vice versa. Such transformation allows system synthesis. Final additive impact of the vector of communications performance indicators on the vector of telematic performance indicators expressed by transformation matrix (see [9] - [12]) can be identified under condition that probability levels of all indicators are set on the same level and performance all indicators are expressed exclusively by time value.

Transformation matrix construction is dependent on the detailed communication solution and its integration into telematic system. Probability of each phenomena appearance in context of other processes is not deeply evaluated in the introductory period. Each telematic element is consequently in several steps evaluated based on the detailed analysis of the particular telematic and communications configuration and its appearance probability in context of the whole system performance. This approach represents subsequent iterative process managed with goal to reach stage where all minor indicators (relations) are eliminated and the major indicators are identified under condition that relevant telematic performance indicators are kept within given tolerance range.

In [13] resented method is designed as broadly as possible with clear aim to be applied in the widest possible range of telematic application. This method can be also successfully used for identification of criteria used to decide, which alternative access technology is evaluated as the best.

2.3 Example of ITS clusters

Transport telematics architecture displays the arrangement of subsystems and functional blocks, including information relationships according to the defined point of view. The task also covers the selection of representative telematics applications ("*cluster*") that shows identical systems requirements.

Among individual representative applications using GNSS (Global Navigation Satellite Systems) the following may be included:

- Securing the movement of means of transport on a transport infrastructure (from the point of view of performance parameters on the GNSS it is a question of securing the accuracy, reliability, availability, integrity, etc., in exactly defined points of the transport infrastructure – the application lays high demands both on the locator proper and the information transmission and processing systems; the solution should comply with the “fail-safe” principle; as typical transport telematics applications we may refer to railway interlocking technology, monitoring the transport of dangerous goods or monitoring the movement of means of transport on an airport area,
- Navigation of the means of transport on a transport network (from the point of view of performance parameters it is a matter of coverage with a signal, time lag at on-line navigation, requirements on exact working maps of the entire geographical area, requirements on speed of information processing both in a mobile unit and the processing centre, requirements on the minimisation of the delay in establishing the position – TTFF - Time to Fix Face); as typical transport telematics applications the following may be referred to: navigation of safety and rescue units to the localised accident place or dynamic or on-line automobile navigation,
- Monitoring and operating the maintenance of transport networks (from the point of view of performance requirements is particularly a matter of an exact transport infrastructure information retrieval, interoperability of individual GIS (Geographical Information Systems) systems of various organisations dealing with maintenance, achievement of high statistical accuracy in establishing a position); as typical transport

telematic applications the following may be referred to: mapping the river channel by means of a measuring ship or measuring the carriageway parameters by means of special measuring vehicles,

- Monitoring the movement of persons and goods on a transport infrastructure (from the point of view of performance requirements it is a matter of transmission and central processing of large amount of information from resources with various accuracy, fast identification of individual sub-sets of the objects of transport, sophisticated information processing in the centre, for instance, the “Floating Car Data”); as typical transport telematic applications the following may be referred to: the use of taxi cabs, public transport passenger vehicles or other utility vehicles equipped with the GNSS systems for traffic flow modelling or the use of localised mobile telephones for modelling the mobility of persons,

- Transport infrastructure charging according to its utilisation (from the point of view of performance parameters it is a matter of reliability, integrity and time lag because the GNSS system is used for the calculation of the amount of the charge and, furthermore, the application places demands on the “fail-safe” principle in terms of the distance covered – if there is an uncertainty about correct charging of the driver, the distance covered is not taken account of); as typical transport telematic application we may give the electronic charging of the transport infrastructure according to vehicle parameters and distance covered.

As a follow-up to the completed analysis and decomposition of performance parameters to individual subsystems a table may be obtained containing performance requirements of above mentioned representatives on the locator proper, telecommunications environment or the information processing centre [2, 3].

The next step following the architecture design is a cluster analysis of individual requirements on individual subsystems of transport telematics chain, including the locator, according to pre-defined criteria. The selection of criteria makes a substantial part of the design because if the architecture is to play an integrative and optimisation role it is necessary to look for stabile optimisation criteria, for instance, the selection of the most exacting criteria of all the representative applications, weighted average of all the most exacting criteria, etc. In this part it is necessary that individual transport telematics applications, their prospective introduction, etc., be heuristically assessed.

The ITS architecture of transport telematics applications based on GNSS results in the concept of space distribution of individual subsystems so that the representative transport telematics applications satisfies the established performance parameters, the infrastructure is utilised as efficient as possible and the in-vehicle mobile unit or the mobile unit on the object of transportation is able to deal with a whole spectrum of transport telematics applications (existing and future).

The selection of advantageous variant of architecture, infrastructure or the locator proper is dominated by systems analysis of requirements of individual representatives of telematics applications.

The presented methodology was used within the solution of RaD project supported by Ministry of Transport of the Czech Republic 802/210/112 "Involvement of the Czech Republic into Galileo Project". The pilot application was chosen with respect to performance clusters as follows [7, 11, 20]:

- Dangerous goods monitoring based on GNSS
- Monitoring of transport means on airport surface with GNSS
- Railway interlocking system using GNSS
- Floating car data collection based on GNSS

All above mentioned projects ware practically realized and performance parameters tested under real conditions.

2.4 ITS market packages

ITS architecture is main basis for the construction of market packages. Each market package defines a group of subsystems, terminators, and data links (logical and physical) dedicated to cover functions directly coming form these elements. Therefore market packages focused on the e.g. traffic data collection, data processing, Park&Ride or public transport services are defined. Basic market package sets are as follows:

- Transport management
- Management of integrated and safety systems
- Traffic information
- Public transport
- Commercial vehicles management
- Data management and archiving
- Advanced vehicle safety systems

The relation between ITS architecture, ITS market packages and real applications is shown on Fig.3.

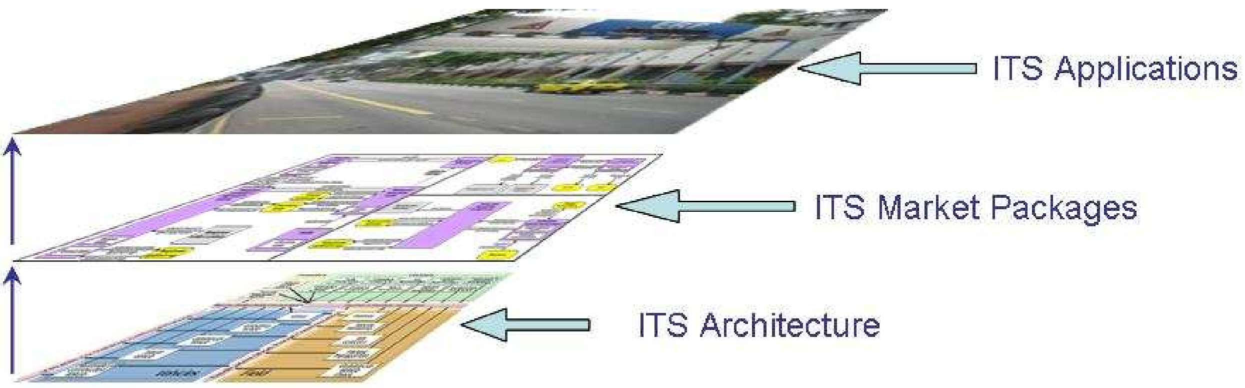

Fig.3 ITS architecture, ITS market packages and real ITS applications

From the system sciences point of view functionally decomposition (or better said functional re-grouping) of the ITS architecture was done. It is possible to identify (among others) through the multiple appearance of its elements throughout defined market packages. However, this redundancy produces possibilities to catch complicated synergy effects, which can occur in particular applications. Undisputed advantage is also the contingency to evaluate ITS application not only like a whole. It represents a way towards evaluation and comparison of various market packages combinations, which are potentially suitable to solve problem given.

Within the context of hardware implementation, each market package is describable as a goal-directly defined group of hardware and software tools. These represent different technologically-implementation means to ensure market package function achievement. In practice, here can be traffic detectors, on-board units, dedicated short-range communication beans, means of satellite communication, information systems, digital maps etc.

3. Mathematical tools for ITS modelling

Presented mathematical tool covers the methodology for estimation of performance parameters, dynamical model identification, fuzzy-linguistic approximation, classification and modelling of large-scale systems by complex-multi-models, methods of data reduction, fusion and comparison.

With help of mathematical tools the appropriate telecommunication environment can be statically/dynamically selected or switched [13], data can be pre-processed [7, 12] and reduced in on-board unit [8], etc. and the ITS technical design can be optimized [2, 4, 19].

3.1 Performance parameters estimation

Let us assume having a normally distributed set of *n* measurements of performance parameters $\mu_{a,1}, \mu_{a,2}, ..., \mu_{a,n}$. If the mean value or standard deviation is not known we can estimate both the mean value $\overline{\mu}_a$ and standard deviation s_a from the measured data as follows:

$$\overline{\mu}_a = \tfrac{1}{n}\sum_{i=1}^{n} \mu_{a,i} \, j$$
$$s_a = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}\left(\mu_{a,i} - \overline{\mu}_a\right)^2} \qquad (1)$$

Let *n* be non-negative integer, α, β are given real numbers $(0 < \alpha, \beta < 1)$ and let $\mu_{a,1}, \mu_{a,2}, ..., \mu_{a,n}, \mu_{a,y}$ be *n+1* independent identically distributed random variable.

Tolerance limits $L = L(\mu_{a,1}, \mu_{a,2}, ..., \mu_{a,n})$ and $U = U(\mu_{a,1}, \mu_{a,2}, ..., \mu_{a,n})$ are defined as values such that the probability is equal to β that the limits include at least a proportion $(1-\alpha)$ of the population. It means that such limits L and U satisfy:

$$P\{P(L < \mu_{a,y} < U) \geq 1 - \alpha\} = \beta \tag{2}$$

A *confidence interval* covers a population parameters with a stated confidence. The *tolerance interval* covers a fixed proportion of the population with a stated confidence. *Confidence limits* are limits within which we expect a given population parameter, such as the mean, to lie. *Statistical tolerance limits* are limits which we expect a stated proportion of the population to lie.

For purpose of this chapter we will present only results derived under the following assumptions:

- $\mu_{a,1}, \mu_{a,2}, \ldots, \mu_{a,n}, \mu_{a,y}$ are *n+1* independent normally distributed random variables with the same mean μ_0 and variance σ_0^2 (equivalently $\mu_{a,1}, \mu_{a,2}, \ldots, \mu_{a,n}, \mu_{a,y}$ is random sample of size *n+1* from the normal distribution with mean μ_0 and variance σ_0^2).
- The symmetry about the mean or its estimation is required.
- The tolerance limits are restricted to the simple form $\overline{\mu}_a - k \cdot s_a$ and $\overline{\mu}_a + k \cdot s_a$, where *k* is so called *tolerance factor*, $\overline{\mu}_a$ and s_a are *sample mean and sample standard deviation*, respectively, given by (1)

Under the above given assumptions condition (2) can be rewritten as follows

$$P\left\{\Phi\left(\frac{U - \mu_0}{\sigma_0}\right) - \Phi\left(\frac{L - \mu_0}{\sigma_0}\right) \geq 1 - \alpha\right\} = \beta \tag{3}$$

where Φ is the distribution function of the normal distribution with mean zero and standard deviation equal to one:

$$\Phi(u) = \frac{1}{\sqrt{2 \cdot \pi}} \int_{-\infty}^{u} e^{-\frac{1}{2} \cdot t^2} dt \tag{4}$$

The solution of the problem to construct tolerance limits depend on the level of knowledge of the normal distribution, i.e. on the level of knowledge of mean $\overline{\mu}_a$ and standard deviation s_a. The variant of *unknown mean value and standard deviation* is the most important in many practical cases but the solution is theoretically very difficult. But fortunately a lot of approximation forms exist based on which the practical simulation could be feasible. We start by task description

$$P\{P[\overline{\mu}_a - k \cdot s_a \leq \mu_{a,y} \leq \overline{\mu}_a + k \cdot s_a] \geq (1 - \alpha)\} = \beta \tag{5}$$

where the sample mean value $\overline{\mu}_a$ and sample standard deviation s_a are estimated from *n* samples according to (2). Howe [15] defines a very simple approximation form for *k*:

$$k \approx \left(\frac{n+1}{n}\right)^{\frac{1}{2}} \cdot z_{\left(1-\alpha/2\right)} \cdot \left(\frac{n-1}{\chi^2_{(1-\beta)}(n-1)}\right)^{\frac{1}{2}} \tag{6}$$

The presented methodology was practically used in [5, 7, 11, 14].

3.2 Bayesian identification methodology

The modellers are able to provide several relationships between the observed data and relevant past. A possible model can be given by ARMA (SISO) modelling for which the pdf (probability density function) of parameterized model output and pdf of unknown parameters is:

$$\begin{aligned} & p(y(n) \mid u(n), D(n-1), \boldsymbol{\Theta}), \\ & p(\boldsymbol{\Theta} \mid u(n), D(n-1)) \end{aligned} \tag{7}$$

$D(n-1)$ means data up to time *n-1* and $\boldsymbol{\Theta}$ vector of unknown parameters. The unknown parameters could be eliminated

$$p(y(n) \mid u(n), D(n-1)) = \int p(y(n) \mid u(n), D(n-1), \boldsymbol{\Theta}) \cdot p(\boldsymbol{\Theta} \mid u(n), D(n-1)) d\boldsymbol{\Theta} \tag{8}$$

under the natural condition of control by e.g. Bayesian methodology [14].

The best output prediction is expressed in probability form as $p(y(n) \mid u(n), D(n-1))$. Pair of input data sample $u(n)$ and output signal $y(n)$ build up the data vector d(n) for time *n*:

$$d(n) = [y(n)\ u(n)] \tag{9}$$

Observation of input-output-pair lead to the history of the system's behaviour

$$d[...n) = [d(0), ..., d(n-2), d(n-1)] \tag{10}$$

and the history including current values is

$$d[...n] = [d(0) ......\ d(n-1)\, d(n)] \tag{11}$$

The parameters $\boldsymbol{\Theta}$ of the model that describe the system S are unknown and are supposed to be independent on time *n* and than we can denote:

$$\boldsymbol{\Theta} = \begin{pmatrix} a_1 \\ . \\ a_m \\ b_0 \\ . \\ b_m \end{pmatrix}, \qquad \boldsymbol{\varphi}(n) = \begin{pmatrix} y(n-1) \\ . \\ y(n-m) \\ u(n) \\ . \\ u(n-m) \end{pmatrix} \tag{12}$$

The model of linear time invariant system can be re-arranged as

$$y(n) = \boldsymbol{\Theta}^T \boldsymbol{\varphi}(n) + e(n) \tag{13}$$

Assuming the noise as being of normal distribution, we can write a model in terms of probabilities for the observed output

$$p(y(n) | \boldsymbol{\Theta}, \boldsymbol{\varphi}(n)) \approx N(\Theta^T \boldsymbol{\varphi}(n), \boldsymbol{\sigma}^2) \tag{14}$$

where $N(\mu, \sigma^2)$ is normal distribution with the mean value μ and standard deviation σ. When expanding the one-dimensional normal distribution to the n-dimensional case, we get the distribution in time *n* :

$$N(\mathbf{\Theta}_n, \mathbf{P}_n) \propto \exp\left(-\frac{1}{2}\left(\mathbf{\Theta}^T \mathbf{P}_n^{-1} \mathbf{\Theta} - 2\mathbf{\Theta}^T \mathbf{P}_n^{-1} \mathbf{\Theta}_n\right)\right) \tag{15}$$

where $\mathbf{\Theta}_n$ denotes the estimate of parameters $\mathbf{\Theta}$ in time n and $\mathbf{P}_n$ represents the covariance matrix of the found parameter estimates. Carrying out the matrix manipulations in the argument of the Euler function also leads to a part independent of $\mathbf{\Theta}$ which we can combine together with the multiplicative constant in another constant and therefore replace the equals by a proportional sign. Symbol $\propto$ means the equality up to the constant.

The probability distribution $p(\mathbf{\Theta}|d[...n))$ of the parameters for time *n* can be recursively calculated according to Bayes's rule:

$$p(\mathbf{\Theta}|d[..n]) \propto p(y(n)|u(n), \mathbf{\Theta}, d[..n)) \cdot p(\mathbf{\Theta}|d[...n)) \tag{16}$$

where $p(\mathbf{\Theta}|d[...n])$ and $p(\mathbf{\Theta}|d[...n))$ are described as derived above:

$$\begin{aligned} p(\mathbf{\Theta}|d[...n]) &\approx N(\mathbf{\Theta}_n, \mathbf{P}_n), \\ p(\mathbf{\Theta}|d[...n)) &\approx N(\mathbf{\Theta}_{n-1}, \mathbf{P}_{n-1}) \end{aligned} \tag{17}$$

The resulting solution of the above equations is in accordance with the *Kalman Filter* [6, 14]:

$$\begin{aligned} \mathbf{P}_n &= \mathbf{P}_{n-1} - \frac{\mathbf{P}_{n-1} \cdot \varphi(n) \cdot \varphi^T(n) \cdot \mathbf{P}_{n-1}}{\sigma^2 + \varphi^T(n) \cdot \mathbf{P}_{n-1} \cdot \varphi(n)} \\ \mathbf{\Theta}_n &= \mathbf{\Theta}_{n-1} + \frac{\mathbf{P}_{n-1} \cdot \varphi(n)}{\sigma^2 + \varphi^T(n) \cdot \mathbf{P}_{n-1} \cdot \varphi(n)} \left(y(n) - \varphi^T(n) \cdot \mathbf{\Theta}_{n-1}\right) \end{aligned} \tag{18}$$

The presented methodology was practically used in [7, 11, 18].

3.3 Complex probabilistic multi-models for large-scale systems

The physical interpretation of complex models will be demonstrated on the example of two binary time systems with outputs $y_1 \in \{0,1\}, y_2 \in \{0,1\}$ as illustrated in Fig. 4.

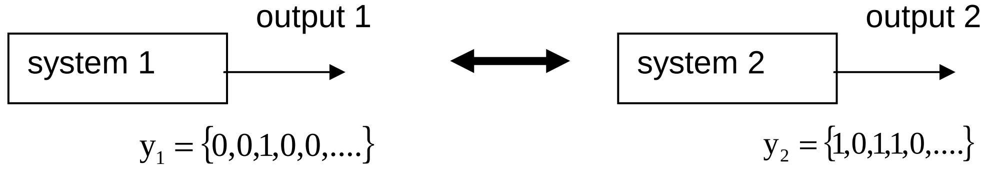

Fig.4 Binary outputs of two systems

Let us define two independent observers marked as H1 and H2 and suppose that each observer registers and makes a model for each different pair of combinations $\{y_1, y_2\}$. The first observer, H1, registers (filters) only the pairs $\{0,0\}$. The second one registers (filters) only the pairs $\{0,1\}$. We speak about "filtration" because the observation process is practically decomposed (filtered) into two independent observation processes.

Observers H1 and H2 create two models based on their observed piece of information. Observer H1 can obtain the joint probability $p(y_1 = 0, y_2 = 0 | H_1)$. Observer H2, on the other hand, can obtain the joint probability $p(y_1 = 0, y_2 = 1 | H_2)$. Let us suppose that due to the observation error (measurement equipments, measurement environment or context) the observer H1 registers in one time interval occurrence of the pair $\{0,0\}$ and the observer H2 registers in the same time interval occurrence of a different pair $\{0,1\}$. This situation can easily occur because the observers H1 and H2 are separated and the exclusivity of their observation is lost. By exclusivity we mean that observers must register either the pair $\{0,0\}$ or $\{0,1\}$ but in no case both of them.

Because observers H1 and H2 do not mutually consult in their observations, it is possible that two different pairs could be registered at the same time. If the number of common registration is not eligible, the observation results (joint probability functions) cannot be calculated according to the well-known probabilistic principles shown in Fig.4 (the sum of joint probabilities obtained by observers H1 and H2 can be even higher than one). The loss of exclusivity is marked by letter C meaning the context dependability. Then the bound models of the two observers mentioned above can be summarized as $p(y_1 = 0, y_2 = 0, H_1 | C)$ and $p(y_1 = 0, y_2 = 1, H_2 | C)$ under the loss of exclusivity condition. To estimate the probability function $p(y_1 = 0)$ from (non exclusive) observations H1 and H2 represented by joint probabilities $p(y_1 = 0, y_2 = 0, H_1 | C)$ and $p(y_1 = 0, y_2 = 1, H_2 | C)$, the phase parameter $\beta_{1,0}$ must be introduced:

$$\begin{aligned} p(y_1 = 0) &= p(y_1 = 0, y_2 = 0, H_1 | C) + p(y_1 = 0, y_2 = 1, H_1 | C) \pm \\ &\pm 2 \cdot \sqrt{p(y_1 = 0, y_2 = 0, H_1 | C) \cdot p(y_1 = 0, y_2 = 1, H_1 | C)} \cdot \cos(\beta_{1,0}) \end{aligned} \tag{19}$$

The phase parameter $\beta_{1,0}$ models the hidden dependency between (non exclusive) observation processes H1 and H2. In other words, the phase parameter removes the overlapping pieces of information caused by decomposition (filtering) of the observation process.

Let the sequence with *m* output values $Y_z, z \in \{1,2,...,m\}$ be represented by a set of *n* models $P(Y_z | H_i), i \in \{1,2,..,n\}$ and let the models be changed over with probability $P(H_i)$. Then, according to well-known Bayes' formula, the probability of z-th output value can be computed as follows:

$$P(Y_z) = \sum_{i=1}^{n} P(Y_z | H_i) \cdot P(H_i) \ . \tag{20}$$

Equation (20) holds only if we know both probabilities $P(H_i)$ and the model components $P(Y_z|H_i), i \in \{1,2,..,n\}$. Model components $P(Y_z|H_i)$ represent, in our approach, the partial knowledge of the large-scale system.

In practical situations the number of model components *n* is finite and is often chosen as a predefined set of multi-model components $P(Y_z|H_i, C)$ where C denotes that the model component is conditioned on designer decision. The probabilities $P(H_i)$ mean the combination factors of the model components. In the case where the real model components $P(Y_z|H_i)$ are the same as the designer's models $P(Y_z|H_i, C)$, the equation (20) is fulfilled. In other cases, the Bayes's formula must be changed so that the designer's decision is omitted (context transition C).

With respect to the inspiration of results achieved in quantum mechanics by [16] the Bayes's formula (20) could be rewritten as follows:

$$P(Y_z) = \sum_{i=1}^{n} P(Y_z|H_i, C) \cdot P(H_i) + 2 \cdot \sum_{k<L} \sqrt{P(Y_z|H_k, C) \cdot P(H_k) \cdot P(Y_z|H_L, C) \cdot P(H_L)} \cdot \lambda_{k.L}^{(z)} \quad , \qquad (21)$$

where coefficients $\lambda_{k,L}^{(z)}$ are normalized statistic deviations that arise due to the designer's decision-making:

$$\lambda_{k,L}^{(z)} = \frac{\frac{1}{n-1}\left(P(H_k) \cdot \left(P(Y_z|H_k) - P(Y_z|H_k, C)\right) + P(H_L) \cdot \left(P(Y_z|H_L) - P(Y_z|H_L, C)\right)\right)}{2 \cdot \sqrt{P(Y_z|H_k, C) \cdot P(H_k) \cdot P(Y_z|H_L, C) \cdot P(H_L)}} \,. \qquad (22)$$

If the designer's decision is right the coefficients $\lambda_{k,L}^{(z)}$ drop to zero and equation (21) converges into equation (20). Parameters $\lambda_{k,L}^{(z)}$ define the angles $\lambda_{k,L}^{(z)} = \cos(\beta_{k,L}^{(z)})$. The parameters $\lambda_{k,L}^{(z)}$ characterize small dependencies (interactions) between the designer's models and is significant, e.g. in the quantum mechanical world.

The presented methodology for large-scale system modeling was published together with the algorithms in [6, 14].

3.4 The methodology of data reduction

By reducing the feature vector's dimensionality the tool of truncated SVD is employed. Let us define the feature matrix **D** composed of the available training data $\mathbf{x}_1^{\omega}, ..., \mathbf{x}_{N_\omega}^{\omega}$:

$$\mathbf{D} = \begin{bmatrix} \mathbf{x}_1^1 \\ . \\ \mathbf{x}_{N_1}^1 \\ . \\ . \\ \mathbf{x}_1^C \\ . \\ \mathbf{x}_{N_C}^C \end{bmatrix} \tag{23}$$

where N_ω stands for a sample count in class ω and the individual feature vector $\mathbf{x} \in R^D$ belongs to one of the C mutually exclusive classes. Let $\mathbf{D} \in R^{Q \times D}$ be a matrix of rank min(Q,D). We suppose that all significant feature vectors are included in the training feature's matrix **D**. The logical sub-sequence of this assumption is that the next feature vector must be written as a linear combination of rows of the feature's matrix **D**. The SVD method corrupts the feature's matrix **D** into multiplication of matrices **U**, **S** and **V**:

$$\mathbf{D} = \mathbf{U} \cdot \mathbf{S} \cdot \mathbf{V}' \tag{24}$$

where $\mathbf{U} \in R^{Q \times Q}, \mathbf{V} \in R^{D \times D}$ are the unitary matrices and $\mathbf{S} \in R^{Q \times D}$ is such a matrix that the singular values are on the main diagonal and all components apart from the main diagonal are equal to zero. The matrix **D** has the numerical $\delta -$ rank k if and only if

$$s_1 \geq s_2 \geq \ldots \geq s_k > \delta \geq s_{k+1} \geq \ldots \geq s_D \tag{25}$$

where $s_i, i \in \{1, 2, .., D\}$ are called the singular values of the matrix **D**. Let us define the matrixes $\mathbf{S}_v$ and $\mathbf{S}_n$ as follows:

$$\mathbf{S}_v = \begin{pmatrix} \mathbf{\Sigma}_1 & 0 \\ 0 & 0 \end{pmatrix}, \ \mathbf{S}_n = \begin{pmatrix} 0 & 0 \\ 0 & \mathbf{\Sigma}_2 \end{pmatrix} \tag{26}$$

where $\mathbf{\Sigma}_1 = \mathrm{diag}(s_1, s_2, \ldots, s_k), \ \mathbf{\Sigma}_2 = \mathrm{diag}(s_{k+1}, s_{k+2}, \ldots, s_D)$

According to the $\delta -$ rank matrix **D** could be split up into more and less important parts:

$$\mathbf{D = U\,S\,V' = U\,S_v\,V' + U\,S_n\,V'} \tag{27}$$

The reconstruction error using only the more significant singular values can be defined as follows:

$$e \approx \mathbf{U} \cdot \mathbf{S}_n \cdot \mathbf{V}' \tag{28}$$

The test of error rate is defined for the purpose of this chapter as the inequality:

$$\frac{\| \mathbf{E} \|^2}{\| \mathbf{D} \|^2} < \mathrm{e} \qquad (29)$$

where matrices **E** and **D** are defined:

$$\mathbf{E = U\, S_n V'} \qquad \mathbf{D = U\, S\, V'} \qquad (30)$$

The norms of matrices **E** and **D** are chosen as:

$$\| \mathbf{E} \|^2 = \mathrm{tr}\{\mathbf{E'E}\} = \sum_i \sum_k \mathrm{E}_{ki}\mathrm{E}_{ki} = \sum_{i,k} \mathrm{E}_{ki}^2 = \mathrm{tr}(\mathbf{VS_n\,'U'US_nV'}) = \sum_{i=k+1}^{n} (\mathrm{s}_i)^2 \qquad (31)$$

$$\| \mathbf{D} \|^2 = \sum_{i=1}^{n} (\mathrm{s}_i)^2 \qquad (32)$$

For predefined reconstruction error *e* the matrix **D** could be approximated as:

$$\mathbf{D} \approx \mathbf{US_vV'} \qquad (33)$$

Without any loss of generality we can take away the components that do not actuate on the matrix multiplication and therefore the equation (3.16) could be modified as:

$$\mathbf{D} \approx \tilde{\mathbf{U}} \cdot \left(\mathbf{S_v} \cdot \mathbf{V}'\right) \qquad (34)$$

where $\tilde{\mathbf{U}}$ is a $Q \times k$ part of the matrix **U** and $(\mathbf{S_v} \cdot \mathbf{V}') \in \mathrm{R}^{k \times D}$.

Let us mark the row of the matrix $\tilde{\mathbf{U}}$ corresponding to the feature vector $\mathbf{x}_i^{\omega}$ by $\mathbf{z}_i^{\omega}$. The reduced feature vector $\mathbf{z}_i^{\omega}$ could be computed from the real feature vector $\mathbf{x}_i^{\omega}$ by the LSE (least square estimation) method:

$$\begin{aligned} \mathbf{z_i^{\omega}} &= \mathbf{x_i^{\omega}} \cdot \mathbf{P} \\ \mathbf{P} &= \left(\mathbf{S_v} \cdot \mathbf{V}'\right)' \cdot \left[\left(\mathbf{S_v} \cdot \mathbf{V}'\right) \cdot \left(\mathbf{S_v} \cdot \mathbf{V}'\right)'\right]^{-1} \end{aligned} \qquad (35)$$

where $\mathbf{P} \in \mathrm{R}^{D \times k}$ is the transformation matrix. The classification or modelling procedure will be applied not to the original feature vector $\mathbf{x}_i^{\omega}$ but to the reduced feature vector $\mathbf{z}_i^{\omega}$.

3.5 Classification and switching problems

Let us define the classification problem as an allocation of the feature vector $\mathbf{x} \in \mathrm{R}^D$ to one of the *C* mutually exclusive classes knowing that the class of **x** takes the value in $\Omega = \{\omega_1, \ldots\ldots, \omega_C\}$ with probabilities $P(\omega_1), \ldots, P(\omega_C)$, respectively, and **x** is a realization of a random vector **X** characterized by a conditional probability density function $\mathbf{p}(\mathbf{x} \mid \omega),\ \ \omega \in \Omega$. This allocation means the selection of best fitted telecommunication technology based on knowledge of **x** vector.

A non-parametric estimate of the ω-th class conditional density provided by the kernel method is:

$$\hat{f}(\mathbf{x} \mid \omega) = \frac{1}{N_\omega \cdot h_\omega^D} \cdot \sum_{i=1}^{N_\omega} K\left(\frac{\mathbf{x} - x_i^\omega}{h_\omega}\right) \tag{36}$$

where $K(\cdot\cdot)$ is a kernel function that integrates to one, h_ω is a smoothing parameter for ω-th class, N_ω stands for sample count in class ω and $\mathbf{x}_1^\omega, ..., \mathbf{x}_{N_\omega}^\omega$ is the independent training data.

It is a well-known fact that the choice of a particular window function is not as important as the proper selection of smoothing parameter. We use the Laplace kernel defined by the following univariate Laplace density function:

$$f_L(x;\mu,\sigma) = \frac{1}{2\cdot\sigma} \cdot \exp\left(-\frac{|x-\mu|}{\sigma}\right) \tag{37}$$

where $x \in R, \mu \in R, \sigma \in (0, \infty)$.

The *product kernel* is used with a vector of smoothing parameters $\boldsymbol{h}_\omega = (h_{1\omega}, ..., h_{D\omega})$ for each class ω. The product kernel density estimate with Laplace kernel is then defined as

$$\hat{f}(\mathbf{x}|\omega) = \frac{1}{N_\omega} \sum_{i=1}^{N_\omega} \prod_{j=1}^{D} \frac{1}{2\cdot h_{\omega,j}} \exp\left(-\frac{|x_j - x_{i,j}^\omega|}{h_{\omega,j}}\right) \tag{38}$$

Smoothing vectors $\boldsymbol{h}_\omega$ are optimized by a pseudo-likelihood cross-validation method using the Expectation-Maximisation (EM) algorithm.

To rank the features according to their discriminative power the standard *between-to within-class* variance ratio is employed. The method is based on the assumption that individual features have Gaussian distributions. The feature vector $\mathbf{x} \in R^D$ takes value to one of C mutually exclusive classes $\Omega = \{\omega_1, ..., \omega_C\}$. The probabilistic measure $Q_{d,i,j}(d, \omega_i, \omega_j)$ of two class separabilities for the feature d (d-th component of feature vector) is defined as

$$Q_{d,i,j}(d, \omega_i, \omega_j) = \frac{\eta \cdot (\sigma_i + \sigma_j)}{|\mu_i - \mu_j|} \tag{39}$$

where ω_i and ω_j are classes and symbol $\eta = 3.0$ denotes the real constant specifying the interval taken into account (probability that observation of normally distributed random variable falls in $[\mu - 3.0 \cdot \sigma, \mu + 3.0 \cdot \sigma]$ is 0.998). The smaller the value of the measure $Q_{i,j,d}$, the better separation of the inspected classes made by the feature *d*. For $Q_{i,j,d} < 1$ both classes are completely separable. The measure is similar to the widely used Fisher criterion.

For multi-class problems, the two-class contributions are accumulated to get a *C*-class separability measure Q(d) for the feature *d* :

$$Q(d) = \sum_{i=1}^{C} \sum_{\substack{j=1 \\ i \neq j}}^{C} Q_{d,i,j}(d, i, j) \tag{40}$$

All the features in the training data are then sorted according to their Q(d) measures. The function Q(d) is similar to a significance measure of the *d*-th component of a feature vector. The subset of *n* first features is selected as an output of this individual feature selection method. The drawback of the method is the assumption of unimodality and the fact that just linear separability is taken into account. On the other hand, the individual feature selection method based on the between-to within-class variance ratio is very fast.

The presented methodology was practically used in [8].

3.6. Fuzzy-linguistic approximation

Application of the linguistic variable fuzzy sets is in the technical branches based on the two main facts:

- Using fuzzy sets is possible to approximate any continuous function (incl. non-linear).
- Through the fuzzy sets it is possible to formalize knowledge represented in the lingual form.

Linguistic variable is such a one where its values are vocables of any natural language. Fuzzy-linguistic variable is then defined as an organized pentad.

$$LV = \{L, T, X, G, M\} \tag{41}$$

where L is name of the variable (e.g. angle, deviation).

T is the set terms (of linguistic values) e.g. small, small positive, big positive.

X represents the universum where terms definitions are given e.g. for angles – {0°, 90°}.

G is a syntactic rule for the term generation

M is a semantic rule – level of term conformity with its meaning.

In the real life, it is not always feasible to have statements in which is impossible to assess clearly their truth and also their implication is not completely accurate as well. Due to this fact so called composition rules are defined in fuzzy approximation to handle this problem:

If R is the fuzzy relation defined on X x Y and A is the fuzzy subset of X (fuzzy set defined on the universum X) then we get the fuzzy set B defined on the universum Y (induced by A) as a composition.

$$B = A \circ R \tag{42}$$

Key rule of the composition rules is the inference one enabling to build up complicated dependencies. The condition is expressed in the form of implication of two or more fuzzy statements.

IF <fuzzy statemnt. 1> AND<fuzzy statemnt. 2> THEN <fuzzy statemnt. 3>

This type of composition rule is also called producing rule.

The presented methodology was practically used in [17].

3.7 Multi-models data fusion

In this chapter methods of multi-models data fusion will be addressed in such a way that the algorithm will enable it to combine information from a set of dynamical models describing the same system. The main goal of data fusion is using the available partial models (together with their expected quality) and combining them in order to find the best model of the original dynamical system.

We expect that there exists a set of dynamical models observing the same parameters of different quality. Each model can process measured data and estimate a model of a partial dynamical system (partial Kalman filtering).

Suppose we have two estimates of the unknown parameter vector $\hat{\mathbf{p}}^{i}(n), \hat{\mathbf{p}}^{j}(n)$ in time *n* based on measured data by *i*-th and *j*-th sensors. Let us mark the difference between the real vector parameter $\mathbf{p}(n)$ and its estimate $\hat{\mathbf{p}}^{i}(n), \hat{\mathbf{p}}^{j}(n)$ as:

$$\begin{aligned} \tilde{\mathbf{p}}^{i}(n) &= \mathbf{p}(n) - \hat{\mathbf{p}}^{i}(n) \\ \tilde{\mathbf{p}}^{j}(n) &= \mathbf{p}(n) - \hat{\mathbf{p}}^{j}(n) \end{aligned} \tag{43}$$

The evolution model of a vector parameter can be described as:

$$\mathbf{p}(n) = \mathbf{A}(n)\mathbf{p}(n-1) + \mathbf{q}(n) \tag{44}$$

where the noise signal $\mathbf{q}(n)$ has the following parameters:

$$E[\mathbf{q}(n)] = \mathbf{0} \qquad \mathrm{var}[\mathbf{q}(n)] = \mathbf{Q}(n) \tag{45}$$

Our two sensors observed the same parameter vector through different measurement models and with different noise conditions. This can be summarized as follows:

$$\begin{aligned} \mathbf{z}^{i}(n) &= \mathbf{D}^{i}(n)\mathbf{p}(n) + \mathbf{w}^{i}(n) \\ \mathbf{z}^{j}(n) &= \mathbf{D}^{j}(n)\mathbf{p}(n) + \mathbf{w}^{j}(n) \end{aligned} \tag{46}$$

where $\mathbf{w}^{i}(n), \mathbf{w}^{j}(n)$ are noise signals on *i*-th and *j*-th sensors with statistical parameters:

$$E\left[\mathbf{w}^i(n)\right] = 0 \qquad \mathrm{var}\left[\mathbf{w}^i(n)\right] = \mathbf{W}^i(n) \tag{47}$$
$$E\left[\mathbf{w}^j(n)\right] = 0 \qquad \mathrm{var}\left[\mathbf{w}^j(n)\right] = \mathbf{W}^j(n)$$
$$\mathrm{var}\left[\mathbf{w}^i(n)\mathbf{w}^j(n)\right] = 0$$

Kalman filtering (dynamical model identification) can be given for each sensor (measuring equipment) in time *n* based on the last measurement in time *n-1* as:

$$\hat{\mathbf{p}}^i(n) = \mathbf{A}(n-1)\cdot\hat{\mathbf{p}}^i(n-1) + \mathbf{H}^i(n)\cdot\left[\mathbf{z}^i(n) - \mathbf{D}^i(n)\mathbf{A}(n-1)\hat{\mathbf{p}}^i(n-1)\right] \tag{48}$$
$$\hat{\mathbf{p}}^j(n) = \mathbf{A}(n-1)\cdot\hat{\mathbf{p}}^j(n-1) + \mathbf{H}^j(n)\cdot\left[\mathbf{z}^j(n) - \mathbf{D}^j(n)\mathbf{A}(n-1)\hat{\mathbf{p}}^j(n-1)\right]$$

where $\mathbf{H}^i(n), \mathbf{H}^j(n)$ are matrices of Kalman gains.

The difference between the estimated and real parameter vectors can be defined:

$$\tilde{\mathbf{p}}^i(n) = \left[\mathbf{I} - \mathbf{H}^i(n)\mathbf{D}^i(n)\right]\cdot\mathbf{A}(n-1)\cdot\tilde{\mathbf{p}}^i(n-1) + \left[\mathbf{I} - \mathbf{H}^i(n)\mathbf{D}^i(n)\right]\cdot\mathbf{q}(n-1) - \mathbf{H}^i(n)\cdot\mathbf{w}^i(n)$$
$$\tilde{\mathbf{p}}^j(n) = \left[\mathbf{I} - \mathbf{H}^j(n)\mathbf{D}^j(n)\right]\cdot\mathbf{A}(n-1)\cdot\tilde{\mathbf{p}}^j(n-1) + \left[\mathbf{I} - \mathbf{H}^j(n)\mathbf{D}^j(n)\right]\cdot\mathbf{q}(n-1) - \mathbf{H}^j(n)\cdot\mathbf{w}^j(n) \tag{49}$$

Joint covariance matrix for differences (real parameter vector versus estimated vector on the first or second sensor) is defined by recurrent form:

$$\mathbf{S}^{ij}(n) = E\left[\tilde{\mathbf{p}}^i(n)\tilde{\mathbf{p}}^j(n)^T\right] =$$
$$= \left[\mathbf{I} - \mathbf{H}^i(n)\mathbf{D}^i(n)\right]\cdot\left[\mathbf{A}(n-1)\mathbf{S}^{ij}(n-1)\mathbf{A}^T(n-1) + \mathbf{Q}(n)\right]\cdot\left[\mathbf{I} - \mathbf{H}^j(n)\mathbf{D}^j(n)\right]^T \tag{50}$$

with an initial value $\mathbf{S}^{ij}(0) = \mathbf{0}$.

For the data fusion vector $\hat{\mathbf{p}}^{ij}$ Gauss-Markov theorem can be used:

$$\hat{\mathbf{p}}^{ij} = E\left[\hat{\mathbf{p}}^{ij}\right] + \mathbf{S}_{\hat{p}^{ij}\hat{p}^j}\mathbf{S}^{-1}_{\hat{p}^j\hat{p}^j}\left(\hat{\mathbf{p}}^j - E\left[\hat{\mathbf{p}}^j\right]\right) \tag{51}$$

where vector $\hat{\mathbf{p}}^j$ is supposed to be the measured parameter vector and $\hat{\mathbf{p}}^i$ is its mean value vector and the mean value vector of the parameter fusion $\hat{\mathbf{p}}^{ij}$. We can write this idea mathematically:

$$E\left[\hat{\mathbf{p}}^{ij}\right] = \hat{\mathbf{p}}^i \qquad E\left[\hat{\mathbf{p}}^j\right] = \hat{\mathbf{p}}^i \tag{52}$$

The multi-model parameter fusion can be described:

$$\hat{\mathbf{p}}^{ij}(n) = \hat{\mathbf{p}}^i(n) +$$
$$+ E\left[\left(\mathbf{p}(n) - \hat{\mathbf{p}}^i(n)\right)\left(\hat{\mathbf{p}}^j(n) - \hat{\mathbf{p}}^i(n)\right)^T\right]E\left[\left(\hat{\mathbf{p}}^j(n) - \hat{\mathbf{p}}^i(n)\right)\left(\hat{\mathbf{p}}^j(n) - \hat{\mathbf{p}}^i(n)\right)^T\right]^{-1}\left(\hat{\mathbf{p}}^j(n) - \hat{\mathbf{p}}^i(n)\right) \tag{53}$$

where the first mean value can be derived:

$$E\left[\left(\mathbf{p}(n) - \hat{\mathbf{p}}^i(n)\right)\left(\hat{\mathbf{p}}^j(n) - \hat{\mathbf{p}}^i(n)\right)^T\right] = E\left[\tilde{\mathbf{p}}^i(n)\left(\tilde{\mathbf{p}}^i(n) - \tilde{\mathbf{p}}^j(n)\right)^T\right] = \mathbf{S}^i(n) - \mathbf{S}^{ij}(n) \tag{54}$$

The second mean value is:

$$\mathrm{E}\left[\left(\hat{\mathbf{p}}^{i}(n)-\hat{\mathbf{p}}^{j}(n)\right)\cdot\left(\hat{\mathbf{p}}^{i}(n)-\hat{\mathbf{p}}^{j}(n)\right)^{T}\right]=\mathrm{E}\left[\left(\tilde{\mathbf{p}}^{i}(n)-\tilde{\mathbf{p}}^{j}(n)\right)\cdot\left(\tilde{\mathbf{p}}^{i}(n)-\tilde{\mathbf{p}}^{j}(n)\right)^{T}\right]=$$
$$=\mathbf{S}^{i}(n)+\mathbf{S}^{j}(n)-\mathbf{S}^{ij}(n)-\mathbf{S}^{ij}(n)^{T} \tag{55}$$

The result of a multi-model data fusion can be finalized. The multi-model parameter fusion is equal to:

$$\hat{\mathbf{p}}^{ij}(n)=\hat{\mathbf{p}}^{i}(n)+(\mathbf{S}^{i}(n)-\mathbf{S}^{ij}(n))\cdot(\mathbf{S}^{i}(n)+\mathbf{S}^{j}(n)-\mathbf{S}^{ij}(n)-\mathbf{S}^{ij}(n)^{T})^{-1}(\hat{\mathbf{p}}^{j}(n)-\hat{\mathbf{p}}^{i}(n)) \tag{56}$$

where the covariance matrix of the multi-model parameter fusion is equal to:

$$\mathbf{M}^{ij}(n)=\mathbf{S}^{i}(n)-\left[\mathbf{S}^{i}(n)-\mathbf{S}^{ij}(n)\right]\cdot\left[\mathbf{S}^{i}(n)+\mathbf{S}^{j}(n)-\mathbf{S}^{ij}(n)-\mathbf{S}^{ij}(n)^{T}\right]^{-1}\left[\mathbf{S}^{i}(n)-\mathbf{S}^{ij}(n)\right]^{T} \tag{57}$$

Equations (56) and (57) represent an algorithm of how to combine the two parameter estimates obtained from different sensors. This data fusion has a lot of applications in practice, e.g. multi-sensor radar data fusion, etc.

The presented methodology was practically used in [11, 14, 18].

3.8 Multi-models data comparison

The data fusion was used to increase the accuracy of vector of the estimated parameters (maximal incorporation of information known from sensors). In a lot of applications the accuracy of the estimate is not as important as other performance parameters, such as integrity, reliability, continuity, etc. In this chapter the method of multi-model comparison will be described in such a way that the significant differences (on predefined probability level) between the data measured by different sensors will be identified and the user will be informed about such system behaviour.

Let us suppose that hypothesis H_o means that sensors *i* and *j* represent the data of the same vector of an unknown parameter or that both sensors are in order and measure the same values with regard to the error of sensors. On the other hand, the hypothesis H_1 means that each of the sensors *i* and *j* measures a different vector of parameters or that one of the sensors is out of order and provides non-correct data.

Multi-model data comparison as a basic method of measured data integrity assessment can be tested with help of $\chi^2(N)$ distribution where N is parameter vector dimensionality. Based on pre-processed measured data (an estimation of a vector of unknown parameters together with its covariance matrix on each sensor), let us define parameter:

$$d(n)=\left(\hat{\mathbf{p}}^{i}(n)-\hat{\mathbf{p}}^{j}(n)\right)\left(\mathbf{S}^{i}(n)+\mathbf{S}^{j}(n)-\mathbf{S}^{ij}(n)-\mathbf{S}^{ij}(n)^{T}\right)^{-1}\left(\hat{\mathbf{p}}^{i}(n)-\hat{\mathbf{p}}^{j}(n)\right)^{T}\leq G \tag{58}$$

where G is known as the *measurement criteria* and it is defined based on the predefined probability level δ (the accepted probability that both *i* and *j* sensors provide correct information but the error is alerted):

$$P[d(n) > G \mid H_0] \leq \delta \tag{59}$$

A typical value of accepted probability is $\delta = 0.05$.

Now we can suppose N sensors data available where the probability of right error detection is marked as P_{RD} and the probability of non-correct error detection as P_{FD}. Because of the enormous safety and economical impact in case of non-correct error alert the method of filtering *"M from N"* will be presented.

Let us have N sensors and for simplicity let us suppose the same probabilities of correct P_{RD} and non-correct error detection P_{FD} on each sensor. If this assumption is not fulfilled the method could be easily extended to a more general case.

As mentioned above the hypothesis H_0 represents the perfect system behaviour (non system error, no sensors error) and hypothesis H_1 a state with detected error (error of system or error of sensors).

In the next equation the probability of error detection on *k* sensors of *N* sensors (*N-k* sensors do not detect errors) is given in case the system does not display any error (conditioned by hypothesis H_0):

$$P[k \mid H_0] = \binom{N}{k} \cdot P_{FD}^{k} \cdot (1 - P_{FD})^{N-k} \tag{60}$$

In same way the probability of error detection by k of N sensors is given in case the system is in an error state (conditioned by hypothesis H_1):

$$P[k \mid H_1] = \binom{N}{k} \cdot P_{RD}^{k} \cdot (1 - P_{RD})^{N-k} \tag{61}$$

The main idea of "M from N" filtering is in selection of value M (threshold) defining the minimum number of sensors that detected error. If M sensors detect error then this error is taken as the real system error and the system starts sending error alert signals. The threshold M should be selected with respect to following probabilities:

$$P_F = \sum_{k=M}^{N} \binom{N}{k} \cdot P_{RD}^{k} \cdot (1 - P_{RD})^{N-k} \tag{62}$$

$$P_D = \sum_{k=M}^{N} \binom{N}{k} \cdot P_{FD}^{k} \cdot (1 - P_{FD})^{N-k}$$

where P_F, P_D means probability of a *false alert* (an error is detected but the system works without any errors) and the probability of *right detection* (the system error is correctly detected).

The number of detectors N and the threshold M can be chosen based on sensors parameters P_S, P_{CH} and required probabilities P_F, P_D.

Methods of multi-model data fusion and comparison are the main tools for estimation of system performance parameters (accuracy, reliability, integrity, continuity, etc.) and can be used for a derivation of an exact definition of false alert and right detection probabilities.

The presented methodology was practically used in [7, 14].

4. ITS effectiveness assessment

Nowadays, at the view of intelligent transport system and services expansion there is the most actual to ask if the ITS implementation and operation is beneficial and effective. With reference to a broad spectrum of traffic problems and situations, where ITS can help to sort them out, it is not always a trivial business to find the right answer.

In spite of several ITS evaluation methodological approaches existing by now, it is not possible to consider further development of this field to be finished. This fact comes out from embarrassments that limit usage of these methodologies markedly.

Within an imaginary list of evaluation approaches their in-homogeneity has played a key role. It is represented by a different "depth and width" of the evaluation procedures, which directly excludes the project comparison possibilities on both national and European level. Therefore for the purposes to evaluate "all" the ITS impacts, including socio-economic and qualitative (e.g. willingness to pay for service, feeling of safety, etc.), the fuzzy-linguistic approximation was chosen and incorporated into the evaluation methodology.

This mathematical mechanism sets up the fuzzy relations which can be used for the system description where relations in between inputs and outputs are not known accurately. From this point of view, ITS fulfil this assumption in many cases because e.g. output/input data values are either not known at all or are known in the short time periods only.

4.1. ITS effectiveness definition

Naturally, ITS effectiveness definition is an essential issue therefore there is a strong focus placed on it. After many discussions and opinion conflicts was decided it is not useful to come up with the brand new math construct but to utilize one of well known approaches focused on the investment's assessment. On account of that, internationally reputable methodology of cost-benefit evaluation (CBA) was chosen and connected with the effectiveness definition. Effectiveness values are represented then by:

Net Present Value (NPV):

$$NPV = \sum_{t=0}^{n} \frac{CF_t}{(1+r)^t} \tag{63}$$

where CF_t represents cash-flow in the time period t, r is the discount factor.

Internal Rate of Return (IRR):

$$0 = \sum_{t=0}^{n} \frac{CF_t}{(1+IRR)^t} \tag{64}$$

Profitability Index (NPV/I):

$$NPV/I = \frac{(PV + CF_0)}{(-CF_0)} = \frac{CF_0 + \sum_{t=1}^{n} \frac{CF_t}{(1+r)^t}}{(-CF_0)} \qquad (65)$$

then

$$NPV/I = \frac{\sum_{t=0}^{n} \frac{CF_t}{(1+r)^t}}{(-CF_0)} \qquad (66)$$

where I represents total of investment costs, PV is a present value Pay-off Period..

CBA also takes into account the time factor (evaluation period - through the discount rate) and thereby an appropriate coverage of all activities associated with the implemental and operational ITS application phases. However, it is necessary to point out that CBA algorithm is not ready for use until all application impacts are known.

4.2. ITS Impacts' assessment and evaluation

Generally, we can say that ITS application benefits depend on many different aspects coming from the physical architecture and its influences on the ITS proposals. Following this fact, benefit indicators were defined to allow particular benefits determination. There were defined not only deterministic quantitative indicators but also socio-economic and qualitative ones characterized by explicit level of uncertainty.

In natural contrast to ITS application benefits (from the evaluation point of view) ITS costs have to be assessed in detail as well. By analogy to the benefit indicators' definitions a set of costs ones was proposed. It is possible, through these indicators, to describe ITS application costs on an appropriate detail level and to create the second part of needed background for the final evaluation.

On the negative impact side (cost side) is this procedure, as compared to benefit evaluation, quite simple. It is given by knowledge of investment and operating costs of almost all ITS applications and by existing of no obstructions involved in their enumeration. On the benefit side is the situation much more complicated because benefits have to be expressed in their natural units first and ex-post transformed into a monetary form in the second step. Transforming mechanism was prepared for strategic evaluation (impact values don't have to be known) and feasibility study level as well.

Accommodation of both these evaluation views is possible to achieve using fuzzy-linguistic approximation. Both principles – expert rules definitions above output and input fuzzy sets (for strategic evaluation) and fuzzy sets estimation based on the measured data (for feasibility study level) – enable finding of unknown function model, as a main input into the final effectiveness calculations.

Searched functions are in this case replaced by expert rules which allow to synthesize number of expert processes and to carry out a rigorous estimation of parameters in demand (defuzzyfication). Suggested technique enables a gradual adding of input and output data and to put function shape estimation more precisely in this way. This ensures that expert estimations (expert rules) are tailored to real situation.

In terms of concurrent methodology analysis alternative approaches (e.g. Kalman filters, neural networks etc.) were analysed, but most of these modern methodologies are based on existence of measured times series. Particular model in demand is given then by an estimation of model parameters on the measured data basis. In case of ITS field either data are not available at all (e.g. for synergy effects covering) or are available in a short time periods only (e.g. measurement once a year, etc.). Therefore substitution of measured data by fuzzy sets is correct.

Basic principle of ITS application effectiveness estimation lies in finding of fuzzy-linguistic approximation of unknown function $y=f(x)$ where y means output parameter and x represents input parameter vector defined through the set of rules (given by ITS experts). It is presumed (for parameter estimation using fuzzy-linguistic approximation) that evaluation procedure is done separately for cost and benefits. Separate models for costs and benefits originate that way and represent basic input into the final effectiveness calculations (defined by CBA indicators).

Cost respectively benefit model of ITS application in view is possible to express through unknown function $n=g1(x)$ respectively, $p=g2(x)$ where n resp. p represents total ITS application costs (benefits) connected with defined time interval and x means input values vector (influencing total ITS solution costs). It is feasible to cover by input vector technical requirements and also other information related to the context of solution given (number of enforcement gantries, area size, number of vehicles inside area, etc.).

Example of fuzzy-linguistic approximation of ITS effectiveness is shown on Fig.5.

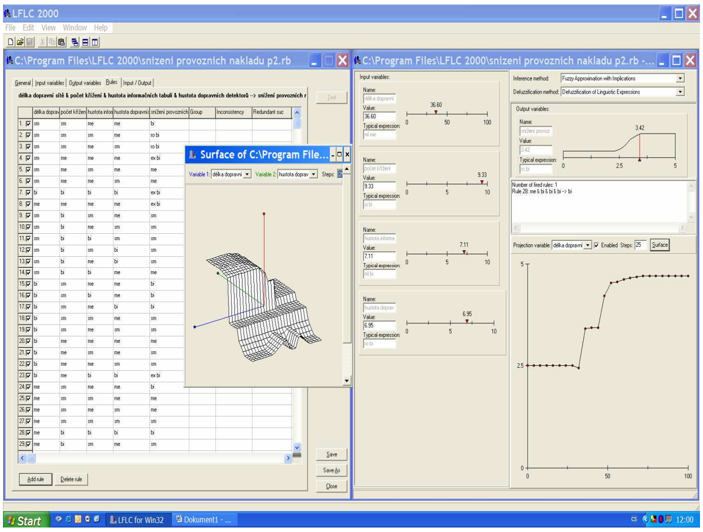

Fig.5. Example of fuzzy-linguistic approximation of ITS effectiveness

In the end of this process initial values of the ITS impacts are ready to be used in the environment of cost-benefit analysis. Here are recalculated into the year cash-flows (using appropriate coefficients) and finally efficiency values are determined.

5. Conclusion

Basic objective of the creation of the ITS design methodology is the achievement of the interoperability between individual ITS applications, including maximum use of available infrastructure by all ITS applications while keeping system requirements in individual ITS applications (technical requirements: safety, reliability, availability, integrity, etc.; transport related requirements: transport comfort, minimisation of external requirements of the transport related process, maintaining transport policy objectives at national and European level, economical requirements: CBA, effectiveness, etc.).

The result of the ITS design methodology should be a design of individual subsystems and functional blocks, including the definition of their system parameters for OBU (On-Board Unit), telecommunication environment and processing centres for all kinds of ITS applications.

However, in the case of various alternatives of the OBU design, transmission environment or processing centres, the system parameters of individual transport telematic applications have to be guaranteed. Correctly conceived design methodology of transport telematic systems in transport organisations will have a direct impact on the following factors:

- Efficient building of telecommunication environment and corporal networks will reduce their expenditures;
- Considerable reduction of transmitted information will reduce expenditures of transmission;
- Definition of requirements from the part of organisations will force the existing operators to offer services with these over-standard requirements, which will result in reduction of expenditures when building special telecommunication environments;
- Economical convenience of new solutions of transmission information will lead to the increase of demand for new technologies of telecommunication networks particularly in the field of access networks;
- It will be possible to secure modular development of ITS in single branches and organisations using the existing systems.

The above factors have an immense impact on economy of building ITS systems. A correctly conceived design methodology, which utilises advanced information processing system, also logically leads to the reduction of information collection and transmission expenditures. The realized pilot projects at Czech Technical University in Prague proved the expectations and yielded into assessment of presented design methodology of ITS systems.

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1992 **Ing.** Faculty of Electrical Engineering, Czech Technical University in Prague (ČVUT) - Radioelectronics

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1996 - 2005 Deputy Head Of Laboratory for System Reliability, Faculty of Transportation Sciences, Czech Technical University in Prague

2000-2005 Vice-president of Association for Transport Telematics in the Czech Republic

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Other activities:

from 2001 - Member of Editorial Board of International Journal Neural Network World

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from 2003 - Member of Scientific Committee of Conference TST (Transport Systems Telematics) , Katowice, Poland

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Scholarship:

1989 Fellowship in Russia and Latvia

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Theory of telematics, intelligent transport systems, applications of mobile communications, modelling of systems and processes, transport telematics, flight radioelectronics, informatics, signals and codes

The selected national and European projects:

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MD 1F44G/092/120 "Economical, ecological and safety electronic fee collection"

MD 1F41E/093/120 "Research of Efficiency of Transport Telematics systems in transportation"

CAPTIVE (Common Application of Traffic Violations Enforcement) - EU Project

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