České vysoké učení technické v Praze
Fakulta elektrotechnická

Czech Technical University in Prague
Faculty of Electrical Engineering

Ing. Jan Faigl, Ph.D.

Multi-Robot Exploration of Unknown Environment

Multi-robotický průzkum neznámého prostředí

Praha, 2014

Summary

Multi-Robot exploration is a probem to acquire information about unknown environment by means of autonomous navigation of a group of mobile robots that use their sensor system to perceive the environment. The fundamental approach to address the exploration is based on an iterative determination where to navigate the robots next, which is decomposed into determination of the suitable goal candidates and selection of the next navigational goal from these candidates. In a case of a multi-robot team, this problem also includes an efficient allocation of the goals among the team members to support cooperation and coordination of the robots' motion, which can be formulated as the task-allocation problem. Moreover, the model of the environment being explored is continuously updated during the exploration mission, therefore it is desirable to continuously utilize the newest information available and perform a frequent replanning of the robots' actions.

New information about the environment can be acquired if the robots travel towards the unexplored parts of the environment, and therefore, goal candidates can be located at the border of the already explored and not yet explored parts of the environment. These locations are called frontiers and various frontier-based approaches have been proposed. The simplest approach based on greedy selection of the closet frontier to the robots has been improved by utility based evaluation that combines expected information gain with the distance cost. In addition, novel approaches to reduce the number of goal candidates have been developed that not only allow to consider more computational demanding assignment procedures, but they also improve the performance for simple greedy based assignments.

On the other hand, several task-allocation algorithms have been deployed in the context of the exploration mission that include standard algorithms from the operational research as well as distributed approaches allowing decision-making under limited communication between the exploring units. However, there is still lack of evaluation methodology to compare different exploration strategies that is not limited to particular experimental setup and that can provide a more general conclusion about the expected performance of the exploration strategy in various exploration missions.

Souhrn

V úloze multi-robotického průzkumu je cílem vytvořit model neznámého prostředí skupinou mobilních robotů vybavených senzory pro snímání svého okolí. Základní přístup řešení této úlohy je založen na opakovaném stanovení množiny možných cílových pozic robotů, ze kterých je možné získat informace o dosud neprozkoumaných částech prostředí. Z těchto možných cílů jsou pak vybírány navigační cíle pro jednotlivé roboty. Součástí úlohy multi-robotikého průzkumu je efektivní rozdělení cílů mezi jednotlivé roboty tak, aby roboty spolupracovaly a zároveň, aby byly jejich pohyby koordinované. V průběhu mise dochází k aktualizaci modelu prostředí, proto je výhodné využít aktuální znalost o prostředí k průběžnému stanovování nových cílů a přeplánování předchozí aktivity.

Nové informace o prostředí lze získat pokud jsou roboty navigovány směrem k neprozkoumaným částem prostředí, proto jsou možné cílové pozice robotů umisťovány na hranici již prozkoumaných a neprozkoumaných částí prostředí. Základní přístup je tak založen na výběru cílů na hranici podle jejich vzdálenosti od robotu. Tento přístup byl rozšířen metodami kombinující jak vzdálenost cíle tak očekávaný přínos k získání nové informace o dosud neprozkoumaných částech prostředí. Kromě toho byly vyvinuty nové metody minimalizující počet možných cílových pozic, které nejen, že vedou na efektivnější průzkum, ale také umožňují místo hladového výběru využít výpočetně náročnější metody přidělování cílů jednotlivým robotům a tím dále zvýšit efektivitu průzkumné mise.

Na druhé straně jsou také rozvíjeny metody přidělování cílů a kromě nasazení standardních algoritmů operačního výzkumu jsou studovány distribuované přístupy, které umožňují řešit úlohu rozhodování za omezené komunikace mezi jednotlivými průzkumnými jednotkami. Kromě těchto výzev je jedním z dalších důležitých problémů robotického průzkumu také návrh metodiky pro hodnocení a porovnání různých strategií průzkumu. Návrh způsobu vyhodnocení, který je nezávislý na konkrétním experimentálním ověření, že daná metoda funguje pro specifický robotický systém, a který umožní poskytnout obecnější závěry o předpokládané efektivitě strategie průzkumu v různých misích a s jiným systémem.

Klíčová slova
Multi-robotický průzkum, robotika, mobilní robotika, plánování, alokace zdrojů

Keywords
Multi-robot exploration, robotics, mobile robotics, planning, task-allocation

Contents

1 Mobile Robot Exploration

Autonomous mobile robot exploration can be considered as an information gathering problem, where one or a group of mobile robots are requested to acquire information about an unknown environment. In robotics, the problem is studied as a problem to create a map of unknown environment by mobile robots, each equipped with a sensor system to perceive its surroundings. Such a map can be further used for planning, support localization, or to find objects of interest located in the environment. Within this context, the exploration is one of the most important robotic problems to address search and rescue missions, where the primary objective is to find eventual victims as quickly as possible.

Regarding this motivation, the exploration can be formulated as a problem to create a map of all reachable parts of an unknown environment and the time to create such a map is the main objective function to measure efficiency of the exploration process. However, the main difficulty of the exploration is that information about the environment is not known in advance and thus a complete exploration plan cannot be prepared prior deployment of the robots in the environment. New information is collected during the mission, and therefore, the decision-making process is a continuous processing of newly acquired sensor measurements to decide where to navigate the robot next.

The fundamental approach to address the exploration problem is based on an iterative determination of the possible goal candidates from which new information about unknown part of the environment can be acquired. In 1997, Yamauchi introduced a frontier based method for a single robot exploration that is based on an iterative assignment of new navigational goal in the next-best-view manner, where the next robot goal is selected from locations that are at the border of already explored and not yet explored parts of the environment, see Fig. 1. He called these candidate locations as frontiers.

Yamauchi proposed to determine frontiers in a grid map of the environment, where each cell is in one of three states: occupied, free, and unknown. Such a grid map can be created from an occupancy grid map that allows a straightforward integration of new sensor measure-

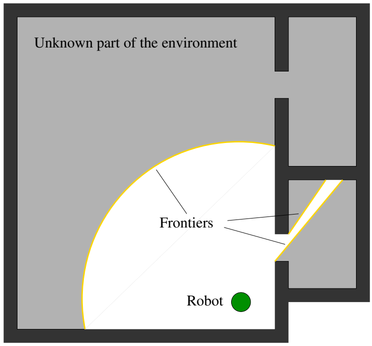


Figure 1: Example of frontiers in the current map of the environment: light gray – unknown parts of the environment; dark gray – obstacles (not yet completely explored); white – explored freespace of the environment; yellow – frontiers; green – robot

ments using Bayes rules [18]. Each cell in the occupancy grid represents a probability the cell is occupied and the grid map is created by thresholding the probability values. Then, frontiers are determined as all freespace cells that are incident with an unknown cell.

One year later, Yamauchi extended this approach for multi-robot missions [24]. The simplicity and straightforward implementation of this method is probably the main reason, why frontier-based approach combined with the grid map representation of the environment becomes very popular in robotics and thus the frontier-based approach is de facto standard approach for the robotic exploration in robotics.

Since the first introduction of the frontier-based exploration, many approaches have been proposed to improve performance of exploration missions. These include approaches that explicitly consider localization of the robot [20], but the main improvement is gained by new strategies to assign the next robot goal. Instead of Yamauchi's greedy approach, researchers proposed utility based evaluation functions that explicitly combine distance of the robot to the goal candidate with the expected information gain [1, 4].

An intuitive way for improving the required time to explore the environment is to use a team of mobile robots. Then, the main problem

Figure 2: Two robots in an experimental evaluation of the multi-robot exploration. The small superimposed map on left is visualization of the current frontiers while on right is shown the the current grid map of the explored part of the environment.

is how to efficiently select the next navigational goals to each robot to perform an efficient coordination and cooperation.

Due to limited information about the environment being explored, a higher frequency of the goal assignment can exploit newer information available, and therefore, the performance of the mission can also be improved by a more frequent decision-making [15]. The computational complexity of the decision-making process depends on the number of robots and also on the number of potential goal candidates. Thus, a selection of the next robot goal can be computationally demanding also for a single robot deployment, e.g., using an evaluation of the candidates based on a solution of the traveling salesman problem (TSP) [26, 16], which is known to be NP-hard unless P=NP. Therefore, a procedure to determine a minimal set of the most promising goal candidates is an important part of the exploration strategy.

It is also worth mentioning the exploration of unknown environment relies on a localization capability of mobile robots, which can be a challenging problem itself. Here, the exploration can be considered within a context of simultaneous localization and mapping [21] and decision making process has to trade-off between exploring new areas and navigation to the previously visited locations to decrease the localization uncertainty. On the other hand, ongoing improvement of the localization techniques using laser range finders allows to consider localization sufficiently precise for indoor structured (e.g., office like) environments [22]. For example this has been reported by several teams participating in CAROTTE[1] multi-robot exploration challenge, where the most important part of the system design was to efficiently share the workload among the team members [6].

These advancements allow to consider new realistic assumptions and provide a ground-work to develop exploration strategies for improving the mission performance not directly related to particular localization capabilities of the used robotic system albeit the localization is still a challenging problem, especially for unstructured and complex outdoor environments where global positioning system cannot be used.

2 Problem Statement

Multi-robot exploration can be considered as an iterative procedure in which new navigational goals are repeatably determined in the current map of the environment and then assigned to the robots. Each robot is autonomously navigated towards the assigned goal and during the motion, sensor measurements of the robot's surroundings are collected to update the map being built. This procedure is repeated until the map of the whole environment is created, which can be indicated by an empty set of the determined goal candidates.

For simplicity, exploration algorithms are usually studied for a homogeneous group of m mobile robots, each equipped with an omnidirectional sensor with the sensing range ρ. Moreover, the robots are considered to move with an average speed and thus the time to explore the environment can be measured as the length of the longest exploration path.

Having a current map of the environment represented as the occupancy grid $\mathcal{O}cc$, a basic schema of the frontier-based exploration can be formalized as follows. Let the current map of the environment be $\mathcal{M}$, a set of determined goal candidates in $\mathcal{M}$ be $\boldsymbol{G} = \{g_1, \ldots, g_n\}$, and the current robot poses be $\boldsymbol{R} = \{r_1, \ldots, r_m\}$. The problem is to

[1] CArtographie par ROboT d'un TErritoire – `http://www.defi-carotte.fr`

assign a goal $g \in \boldsymbol{G}$ for each robot $r \in \boldsymbol{R}$ that will minimize the total required time to explore the whole environment.[2] Thus, *the problem is to determine the goal candidates* $\boldsymbol{G}$ *and select the next navigational goal* $g_j \in \boldsymbol{G}$ *for each robot* $r_i \in \boldsymbol{R}$*:* $\{\langle g_{j_1}, r_1\rangle, \ldots, \langle g_{j_m}, r_m\rangle\}$*, in such a way the total exploration time is minimal*.

Notice that although we aim to minimize the total required exploration time, the decisions are made iteratively using only the available information (the current map) about the environment, which is continuously updated during the mission execution. The main steps of the exploration are:

1. Determine goal candidates using the current map $\mathcal{M}$.
2. Assign the candidates as the next navigational goals of the robots.
3. Navigate the robots towards the goals and update $\mathcal{M}$.

These steps are repeated until $|\boldsymbol{G}| > 0$, which indicates the whole environment is explored. The exploration strategy that is responsible for the decision-making process can be considered as the couple of the procedures at Step 1 and Step 2.

3 Exploration Strategies

3.1 Goal Candidates Determination

The most straightforward method to determine goal candidates in the current map of the environment being explored is to consider all frontier cells as potential goal candidates. However, a huge number of candidates may increase computational burden of the goal assignment, while many of such candidates are clearly not perspective goals. For example, a simple filtration can reduce the number of candidates to consider only frontiers that are not too close to obstacle regions to increase safety of the navigation [15].

Authors [14] considered the problem of determining goal candidates as a variant of the art gallery problem and utilized the sensor placement algorithm [13] as a randomized greedy set coverage

[2]Such a time can be approximated by the maximal travelled distance by an individual robot.

technique to find the best view locations to cover the frontiers organized into single connected components called *free curves*. The goal candidates are randomly placed within the sensor range from the free curves. Then, each candidate q is evaluated using a utility function $A(q)exp(-\lambda L(q))$, where λ is a positive constant, $L(q)$ is the length of the robot–candidate path, and $A(q)$ is the expected maximal area of the unknown part of the environment that can be explored from the candidate q. A similar approach is considered in [19], where distance and utility costs are combined in the same way with an additional exponential term to consider orientation of the sensor at the goal candidate location.

(a) (b)

Figure 3: Examples of determined goal candidates by the methods: (**a**) representatives of the frontiers [16]; (**b**) complete coverage [7]; The unknown part of the environment is in gray, explored freespace is in white and detected obstacle are in black. Free curves (frontier cells) are distinguished by different colors and determined goal candidates are represented as red discs.

Representatives of free curves (frontiers cells) are utilized in [16] to decrease computational burden of the proposed next–goal assignment based on a solution of the TSP in a single robot exploration. The method selects n_r goal candidates for each free curve as the means of n_r clusters determined by the K-means algorithm. The number n_r is determined according to the size of the particular free curve (i.e., the number of the frontiers cells of the curve) and sensor range. An example of such representatives is shown in Fig. 3a.

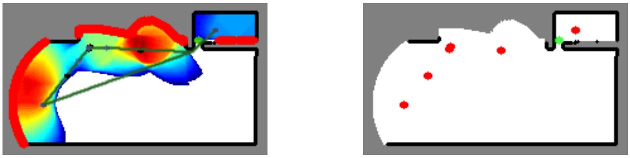
A deterministic procedure to determine a minimal set of goal candidates from which all frontiers cells can be covered is presented in [7]. The paper also provides an extensive comparison with other goal candidates determination methods combined with various assignment procedures. The results indicate that the proposed method (called *complete coverage*) of goal candidates determination provides the best exploration performance. Besides, the results also indicate that a computationally efficient selection of the free edge representatives [16] improves performance of the exploration significantly regardless the assignment method, i.e., it also improves Yamauchi's greedy assignment. Therefore, based on these results it does not make sense to use all frontier cells as potential goal candidates.

The main difference of the complete coverage approach [7] and the previous frontier-based approaches is in determination of the goal candidates that are not frontier cells, but are located deeper in the already explored space, see Fig. 3b. This motivate us to consider the consecutive selection of the next robot goal and we proposed a goal candidates determination method based on a solution of the traveling salesman problem with neighborhoods (TSPN) using a self-organizing map based approach [10]. Early results reported in [11] indicate that combining determination of the goal candidates with a consideration of the further selection of the goal can improve the overall mission performance. However, the current results do not provide a strong evidence of the improvement. A solution of the underlying TSPN that provides a coverage of the frontier cells and the found goal candidates are shown in Fig. 4.

3.2 Goal Assignment Procedures

Following the standard robotic exploration based on the next-best-view approach, the assignment of the goal candidates to a group of mobile robots can be considered as the task-allocation problem [12]. The problem is to find the best assignment of n goals to m robots maximizing the overall utility, i.e., to find one goal for each robot.

In [17], authors use Hungarian algorithm for the assignment of the goal candidates to the robots and Voronoi Graphs of the current known environment to explore a single room by one robot. Hungarian algorithm is an optimal task-allocation algorithm for the given cost matrix in which each cell value is a distance cost of the robot–goal as-



(a) (b)

Figure 4: Example of the TSPN solution to determine a shortest path from which all frontier cells are covered. The unknown part of the environment is in gray and the frontier cells are in red: (**a**) – the color map indicates how many frontier cells can be covered from a particular locations, the red color means a high number of covered frontier cells while blue indicates a low number of covered cells; (**b**) – determined goal candidates from the solution of the TSPN.

signment. The algorithm has complexity $O(n^3)$, where n is the size of the squared cost matrix and it is a suitable technique for centralized approaches where information about all exploring units and a global integrated map of the explored parts of the environment is available.

A distributed assignment algorithm called Broadcast of Local Eligibility (BLE) has been proposed in [23]. A pair $\langle robot, task\rangle$ with the highest utility is considered to assign the task to the robot without tasks. The BLE algorithm works iteratively until each robot has assigned a task; thus, the algorithm is also called iterative assignment.

Another goal assignment strategy that can be used in a distributed environment is the MinPos algorithm proposed by the authors of [5]. The assignment strategy is based on a computation of the rank $r_{i,j}$ for each goal i and robot j. The rank $r_{i,j}$ is the number of robots that are closer to the goal candidate i than the robot j. Then, each robot selects the goal for which its rank is minimal. In a case several goal candidates have the same minimal rank for the robot i, the closest goal candidate to the robot is selected from such candidates. The rank can be simply computed in a centralized way but it can also be computed locally based only on the information about the position of the other robots in the vicinity of the particular robot.

Additional approach to the distributed solution of the goal assignment in multi-robot exploration can be based on market (or auction) based approaches in which a robot (auctioneer) offers a task and other robots bid. If any robot bids with a higher price than the auctioneer's offer, the task is exchanged. This approach has been utilized in [26], where a robot considers its goals in a tour and new (exchanged) goal is inserted into the tour regarding minimization of the tour's cost, i.e., the problem is a variant of the traveling salesman problem (TSP).

A selection of the next navigational goal based on evaluation only a distance cost has been studied in [16]. However, instead of single robot–goal distance, the distance is computed as a length of the shortest path to visit all current goal candidates, and the problem is formulated as a variant of the TSP. The authors call the distance as the TSP distance cost and reported about 30 percentage points shorter exploration path than using the standard greedy approach. A computationally demanding solution of the TSP is addressed by the Chained Lin-Kernighan heuristic [3] to find approximate solution of the TSP with a sufficiently good quality and without expensive computational requirements, which allows to use the method under real-time constraints in a practical deployment.

In [9], we extended the TSP distance cost approach for a single robot to multi-robot exploration and formulate the problem as the multiple traveling salesman problem (MTSP). The MTSP is addressed by the ⟨*cluster first*, *route second*⟩ heuristic and the complete assignment algorithm works as follows. First, the goal candidates are clustered by the K-means algorithm where the clusters are seeded with the robots positions. Then, each cluster is assigned to the particular robot used for the cluster initialization and the next robot goal is determined according to the TSP distance cost to visit all the goal candidates in the cluster. Similarly to [16], the solution of the TSP is solved by the Chained Lin-Kernighan heuristic [3] from the CONCORDE package [2].

A comparison of the aforementioned exploration strategies have been reported in [9, 7, 8]. Regarding the reported results, the most efficient exploration strategies are the Hungarian assignment and MTSP based assignment, example of exploration paths for 3 robots and the same evaluation setup is shown in Fig. 5. Both approaches are central-

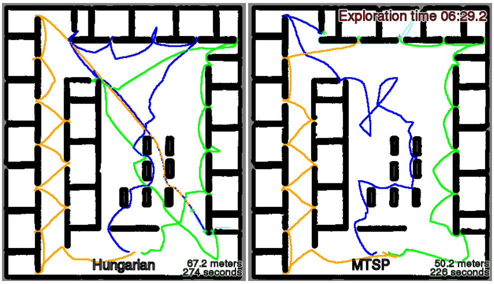


Figure 5: Example of the exploration paths for the same scenario with 3 robots and Hungarian assignment and MTSP-based assignment

ized, and therefore, the BLE algorithm that provides not significantly worse performance than the Hungarian algorithm can be considered for distributed decision-making.

4 Evaluation of Exploration Strategies

Although the robotic exploration is a challenging problem, we also deal with a problem how to compare different exploration strategies. It is because performance of the exploration, e.g., in a term of the required exploration time, depends on many factors and it is influenced by a particular robotic platform and its navigational system including motion control. The most visible factor is a frequency of the decision-making as information about the environment is collected during the exploration mission and thus one can expect that a more frequent re-planning may provide better performance because of new available information.

In robotics, the mission performance of the exploration system is usually considered in a practical deployment, where simple and fast algorithm can actually perform better because of the limited on-board computational power, and therefore, such results and comparisons are less general, especially regarding a more sophisticated approach considering a longer planning horizon that are currently limited by the available computational power. In addition, the decision-making process is stochastic because of underlying sources of uncertainties in robot motion and sensor measurements, and therefore, the expected performance should be measured as statistical indicators. However, practical deployment usually limits the evaluation to several trials that do not provide statistically significantly evidence about the performance. Moreover, and probably more importantly, such an evaluation can also be biased by a tuning of the parameters to the particular robotic system. Therefore, we can identify that even though a practical deployment provides a valuable verification of the particular robotic exploration system, such results and conclusions about the system performance are limited to the particular system and they may not be directly generalized.

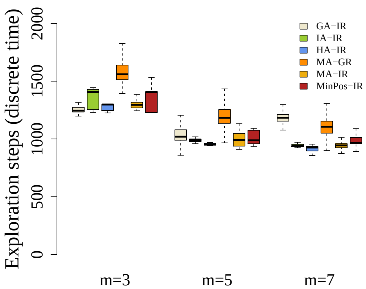
Based on this observation we propose to also evaluate performance of the exploration in a well defined setup that is independent on the available computational resources. We proposed an evaluation framework based on a discrete event based simulator. Regarding the exploration strategy, we can identify three main decision-making parts in the frontier-based exploration approaches. The first part is the method how new goal candidates are determined from the frontier cells in the actual map of the environment. The second important decision-making process is the assignment of the goal candidates to the robots together with the selection of the next navigational goal for each robot. Finally, the decision-making also depends on how often these two parts are repeated, and therefore, the third part is the condition when to perform new assignment. These parts are the main components of the exploration procedure and we call them exploration strategies. The procedure can be summarized as follow:

1. Initialize the occupancy grid $\mathcal{O}cc$ and set the initial plans to $\mathcal{P} = (P_1, \ldots, P_m)$, where $P_i = \{\emptyset\}$ for each robot $1 \leq i \leq m$.
2. `Repeat`
   (a) Navigate robots towards their goals using the plans $\mathcal{P}$, i.e., move each robot to the next cell from the plan;
   (b) Collect new measurements with the range ρ to the occupancy grid $\mathcal{O}cc$;

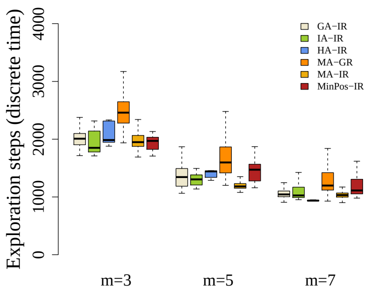
   `Until` **replanning condition is meet**.
3. Update a navigation map $\mathcal{M}$ from the current $\mathcal{O}cc$.
4. Detect all frontiers $\mathcal{F}$ in the current map $\mathcal{M}$.

5. **Determine goal candidates $\boldsymbol{G}$** from the frontiers $\mathcal{F}$.
6. `If` $|\boldsymbol{G}| > 0$ **assign goals to the robot**
   - $(\langle r_1, g_{r_1}\rangle, \ldots, \langle r_m, g_{r_m}\rangle)$=assign$(\boldsymbol{R}, \boldsymbol{G}, \mathcal{M}), r_i \in \boldsymbol{R}, g_{r_i} \in \boldsymbol{G}$;
   - Plan paths to the assigned goals (as sequences of grid cells) $\mathcal{P} = \text{plan}(\langle r_1, g_{r_1}\rangle, \ldots, \langle r_m, g_{r_m}\rangle, \mathcal{M})$;
   - Go to Step 2.
7. Stop all robots (all reachable parts of the environment are explored).

The navigation part (Step 2a and Step 2b) is repeated according to the specified condition. Two basic (limiting) variants of the condition can be distinguished: (1) a robot reaches its goal; (2) a new assignment is performed whenever an assigned goal will no longer be a frontier cell, e.g., a surrounding unknown area becomes explored. We call the first variant as the *goal replanning* (GR) condition and the second variant is called the *immediate replanning* (IR) condition.



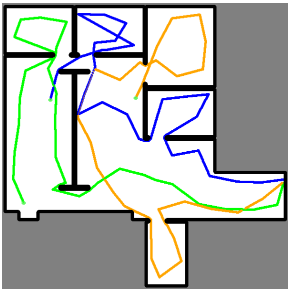
(a) *autolab* environment, ρ=3 m



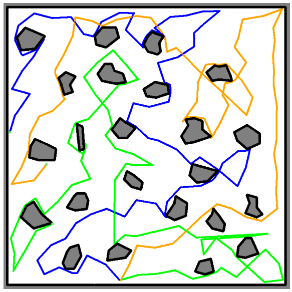
(b) *potholes* environment, ρ=5 m

Figure 6: Required time to explore the *autolab* and *potholes* environments with m robots, sensor range ρ, IR and GR replanning conditions

In this evaluation framework, the robot motion is restricted to traverse a single grid cell per one simulation step, which avoids an influence of the available computational power and it also allows to evaluate performance of computationally demanding exploration strategies independently on the used hardware. Thus, this evaluation framework allows to measure the performance of the exploration as the required exploration steps. Besides, it can be used to evaluate the results statistically using thousands of trials.



(a) *autolab* environment



(b) *potholes* environment

Figure 7: Final exploration paths in the evaluated environments, the number of robots m=3 and sensor range ρ=3 m. Obstacles (showed in black) are enlarged to avoid collisions of the disc shaped robots (with a diameter 0.3 m) with obstacles.

An example of the results provided by the framework that are visualized as the five-number summary from 3780 trials is depicted in Fig. 6. The results show how frequency of replanning can significantly change performance of the computationally demanding MTSP-based assignment (MA) in comparison with greedy approach [25] (GA), iterative assignment (IA) [23], Hungarian assignment (HA) [17], and MinPos assignment [5] accompanied by a modified determination of the goal candidates as representatives of free curves [16]. The particular environments called *autolab* and *potholes* are shown in Fig. 7.

5 Conclusion

A problem of multi-robotic exploration of unknown environment can be addressed by the well established frontier-based approach, where the problem of efficient sharing of the work load among the team members is formulated as the task-allocation problem. Based on the nowadays technological advancements providing robust navigational capabilities in structured environment, we can identify two main research streams in robotic exploration. The first stream aims to address the exploration in a more complex, large-scale unstructured environments, while the second streams aims to improve the performance of the exploration mission based on more sophisticated decision-making processes. The presented approaches follow the second stream.

Regarding multi-robot exploration, we can further identify additional challenges related to practical deployment of the system in different scenarios. These include consideration of efficient information sharing under limited communication, which leads to a local decision-making and distributed algorithms such as the BLE and MinPos algorithms. Another challenging topic is an exploration under dynamic environment that can be further generalized to exploration of some studied phenomena in the environment and thus the goal is not to create a map of the environment, but rather to build a sufficiently precise model of the phenomena.

Although the aforementioned problems are very challenging, there is also one fundamental challenge that need to by address. The challenge is to move the research from a demonstration that a particular approach is working for specific tuned parameters, to a systematical evaluation that will allow to compare different exploration strategies and to provide more general conclusions about the expected system performance under different setups.

6 References

[1] F. Amigoni and V. Caglioti. An information-based exploration strategy for environment mapping with mobile robots. *Robotics and Autonomous Systems*, 58(5):684–699, 2010.

[2] D. Applegate, R. Bixby, V. Chvátal, and W. Cook. CONCORDE TSP Solver, 2003. URL `http://www.tsp.gatech.edu/concorde.html`. [cited 2 May 2013].

[3] D. Applegate, W. Cook, and A. Rohe. Chained Lin-Kernighan for large traveling salesman problems. *Informs J. on Computing*, 15(1):82–92, 2003.

[4] N. Basilico and F. Amigoni. Exploration strategies based on multi-criteria decision making for searching environments in rescue operations. *Autonomous Robots*, 31(4):401–417, 2011.

[5] A. Bautin, O. Simonin, and F. Charpillet. Minpos: A novel frontier allocation algorithm for multi-robot exploration. In *Proceedings of the 5th International Conference on Intelligent Robotics and Applications - Volume Part II*, ICIRA'12, pages 496–508, Berlin, Heidelberg, 2012. Springer-Verlag.

[6] A. Bautin, P. Lucidarme, R. Guyonneau, O. Simonin, S. Lagrange, N. Delanoue, and F. Charpillet. Cart-o-matic project: autonomous and collaborative multi-robot localization, exploration and mapping. In *Proc. of 5th Workshop on Planning, Perception and Navigation for Intelligent Vehicles, organized in conjunction with IROS'13*, pages 210–215, 2013.

[7] J. Faigl and M. Kulich. On determination of goal candidates in frontier-based multi-robot exploration. In *Proceedings of the 6th European Conference on Mobile Robots*, pages 210–215, 2013.

[8] J. Faigl and P. Vaněk. On parameters settings in multi-robot exploration strategies. In *Workshop on Multi-Agent Coordination in Robotic Exploration organized in conjuction of the 21st European Conference on Artificial Intelligence (ECAI'14')*, 2014.

[9] J. Faigl, M. Kulich, and L. Přeučil. Goal assignment using distance cost in multi-robot exploration. In *IEEE/RSJ Int. Conf. Intelligent Robots and Systems (IROS)*, pages 896–901, 2012.

[10] J. Faigl, V. Vonásek, and L. Přeučil. Visiting convex regions in a polygonal map. *Robotics and Autonomous Systems*, 61(10):1070–1083, 2013.

[11] J. Faigl, P. Vaněk, and M. Kulich. Self-organizing map for determination of goal candidates in mobile robot exploration. In *ESANN*, pages 589–594, 2014.

[12] B. P. Gerkey and M. J. Mataric. A formal analysis and taxonomy of task allocation in multi-robot systems. *International Journal of Robotics Research*, 23(9):939–954, 2004.

[13] H. González-Baños and J.-C. Latombe. Planning Robot Motions for Range-Image Acquisition and Automatic 3D Model Construction. In *AAAI Fall Symposium*, 1998.

[14] H. H. Gonzalez-Banos and J.-C. Latombe. Navigation Strategies for Exploring Indoor Environments. *International Journal of Robotics Research*, 21 (10-11):829–848, 2002.

[15] D. Holz, N. Basilico, F. Amigoni, and S. Behnke. Evaluating the efficiency of frontier-based exploration strategies. In *ISR/ROBOTIK*, pages 1–8, 2010.

[16] M. Kulich, J. Faigl, and L. Přeučil. On Distance Utility in the Exploration Task. In *IEEE International Conference on Robotics and Automation (ICRA)*, pages 4455–4460, 2011.

[17] K. M. Wurm, C. Stachniss, and W. Burgard. Coordinated multi-robot exploration using a segmentation of the environment. In *IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS)*, pages 1160–1165, 2008.

[18] H. P. Moravec. Sensor fusion in certainty grids for mobile robots. *AI Magazine*, 9(2):61–74, 1988.

[19] A. Nüchter, H. Surmann, and J. Hertzberg. Planning robot motion for 3d digitalization of indoor environments. In *ICAR*, pages 222–227, 2003.

[20] I. M. Rekleitis, G. Dudek, and E. Milios. Multi-robot collaboration for robust exploration. *Annals of Mathematics and Artificial Intelligence*, 31(1-4):7–40, 2001.

[21] S. Thrun, W. Burgard, and D. Fox. *Probabilistic Robotics (Intelligent Robotics and Autonomous Agents)*. The MIT Press, 2005.

[22] E. Tsardoulias and L. Petrou. Critical rays scan match slam. *Journal of Intelligent & Robotic Systems*, 72(3-4):441–462, 2013.

[23] B. B. Werger and M. J. Mataric. Broadcast of local eligibility for multi-target observation. In L. E. Parker, G. Bekey, and J. Barhen, editors, *Distributed Autonomous Robotic Systems 4*, pages 347–356. Springer-Verlag, 2001.

[24] B. Yamauchi. Frontier-based exploration using multiple robots. In *Proceedings of the second international conference on Autonomous agents*, AGENTS '98, pages 47–53, NY, USA, 1998. ACM.

[25] B. Yamauchi. Decentralized coordination for multirobot exploration. *Robotics and Autonomous Systems*, 29:111–118, 1999.

[26] R. Zlot, A. Stentz, M. Dias, and S. Thayer. Multi-robot exploration controlled by a market economy. In *IEEE International Conference on Robotics and Automation (ICRA)*, pages 3016–3023, 2002.

Ing. Jan Faigl, Ph.D.

Jan Faigl received his PhD degree in Artificial Intelligence and Biocybernetics in 2010 and his master degree in Technical Cybernetics in 2003 both from the Czech Technical University in Prague (CTU). He is a senior research scientist in the Agent Technology Center at the Department of Computer Science. He joined Faculty of Electrical Engineering (FEE) at the CTU in 2003 and he acted as a full-time researcher at the Department of Cybernetics since 2006. During 2003–2006 he also acted as system analyst and programmer at the ProTys, Inc. working on railways safety systems. Since 2013, he is with the Department of Computer Science, FEE, CTU.

He received Antonín Svoboda award for the best dissertation of the year 2010 from the Czech Society for Cybernetics and Informatics. In 2013, he has been awarded Fulbright Scholarship for 6 months research stay at the University of Southern California, in the Robotic Embedded System Laboratory. Jan is an author or co-author of more than 60 publications in international conferences and journals.

He acted as a reviewer for Journal of Intelligent and Robotic System, IEEE Transactions on Cybernetics, Autonomous Robots, Robotics and Autonomous Systems, Applied Mathematics and Computation, and Journal of The Franklin Institute. He also served as a member of program committee for AAAI-14, IEEE International Conference SMC 2014 and he is a member of the program committee for IJCNN 2015.

Teaching experience

Jan is currently teaching *Programming 1* course in Open Informatics and Robotics and Cybernetics study programs. Previously, he taught *Programming methodologies* (2003–2007), *Programming techniques* (2005–2011), *Robots* (2009–2013) and *System Reliability and Total Quality Management* (2003–2010). He participated in introduction of the course *Team work and its organization* (2010–2012). He acts as a chair of the committee for teaching programming at the FEE, CTU.

Jan supervised 17 bachelor and master theses and 1 bachelor thesis and 3 master theses have been awarded the Dean award for the best thesis. He is currently supervising one Ph.D. student.