České vysoké učení technické v Praze
Fakulta jaderná a fyzikálně inženýrská

Czech Technical University in Prague
Faculty of Nuclear Sciences and Physical Engineering

Mgr. Milan Krbálek, Ph.D.

Socio-fyzikální modelování dynamiky dopravního proudu

Socio-Physical Modeling of Traffic Stream Dynamics

Summary

Many contemporary treatises dealing with realistic simulations of vehicular streams discuss possibilities for application of thermodynamic approaches to the traffic modeling. These attempts have been partially successful if the local thermodynamics was adopted. Specifically, introducing the socio-physical particle scheme with mutual interactions described by repulsive forces among the subsequent elements one can obtain surprisingly good analytical estimations for microscopic traffic quantities or their statistical distributions. In spite of the fact that the initial scheme is of thermodynamic substance (contrary to the fact that vehicular traffic is the system driven far from equilibrium) the alternative formulation of the model leads to an interesting insight into the psychological background of traffic interactions.

This habilitation lecture introduces the thermal-like particle ensemble whose intelligent elements are interconnected by the psychological short-ranged interactions and stochastically influenced by the nonzero level of mental strain. The analytically-derived probability densities for individual velocities of agents and clearances among them are discussed with respect to real-road traffic data-samples. The found similarity is utilized in a vicarious detection of mental strain of car drives.

Souhrn

Mnoho současných prací zabývajících se realistickými simulacemi dopravních proudů diskutuje možnosti aplikace termodynamických přístupů v dopravním modelování. Tyto pokusy byly částečně úspěšné zejména v okamžiku, kdy byla aplikována lokální termodynamika. Například vytvořením socio-fyzikálního částicového schématu, v němž jsou vzájemné interakce popsány repulzivními silami mezi sousedními elementy, dostáváme překvapivě kvalitní analytické odhady pro mikroskopické dopravní veličiny, resp. jejich statistická rozdělení. Navzdory faktu, že původní schéma je termodynamické povahy (oproti všeobecně známé skutečnosti, že automobilová doprava je řízeným nerovnovážným systémem), vede alternativní formulace modelu k zajímavému vhledu do psychologického pozadí dopravních interakcí.

Tento habilitační spis diskutuje kvazitermalní částicový soubor, jehož inteligentní elementy jsou propojeny psychologickými krátkodosahovými interakcemi a stochasticky ovlivněny nenulovou hladinou psychického vypětí. Analyticky odvozené hustoty pravděpodobnosti pro rychlosti jednotlivých vozidel či jejich rozestupy jsou diskutovány v kontextu reálných dopravních experimentů. Nalezená podobnost je zužitkována v nepřímé detekci psychického vypětí řidičů.

Klíčová slova: kvantitativní sociodynamika, automobilová doprava, deterministický chaos, celulární modelování, termodynamické modely, statistická rozdělení, fundamentální diagram dopravního toku, dopravní kongesce

Keywords: quantitative socio-dynamics, vehicular traffic, deterministic chaos, cellular modeling, thermodynamic models, statistical distributions, fundamental diagram of traffic flow, traffic jam

České vysoké učení technické v Praze

Název: Socio-fyzikální modelování dynamiky dopravního proudu
Autor: Mgr. Milan Krbálek, Ph.D.
Počet stran: 22
Náklad: 150 výtisků

ISBN 00-00-00000-0

Contents

1 Introduction

Movements of an arbitrary group of humans or animals show many common features originated from group dynamics ([21],[3], and [20]). For purposes of this habilitation lecture the animal/human groups are understood as a self-organized systems whose individual agents are influenced by other agents in the group (see for example [19]). It means that each agent adapts its behavior to the behavior of the rest of group. Such a influence is naturally restricted to the interactions with agents occurring in the close neighborhood (short-ranged interactions). Moreover, the decision-making process of the moving agent is influenced by the various factors (individuality of the agent, actual mental strain, control signals, information inflow, random factors and so on). Typically, the mediated collective decision-making of a group leads to effects of crowding, i.e. to the formation of congested states when the movement of one agent is strongly restricted by other agents. One of these effects is visualized in the figure 1.

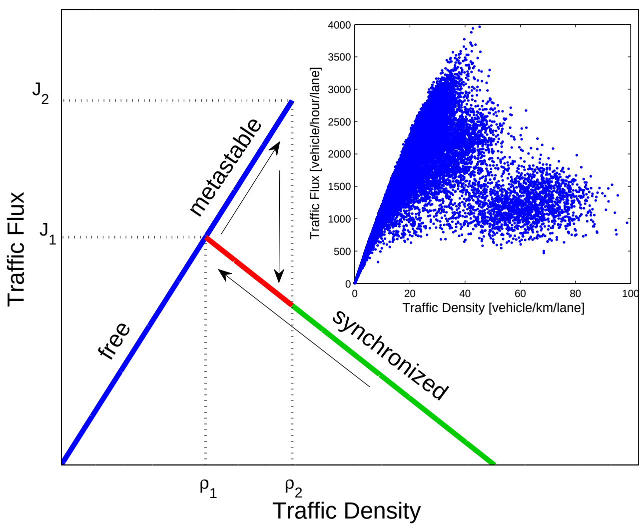

Figure 1: Schematic Representation of Fundamental Diagram of Traffic Flow. Traffic flux J as a function of traffic density ϱ. The diagram is divided into three regions: 1. The free flow region (up to $\varrho = \varrho_1$) where cars move without any restrictions. 2. The region of metastable traffic ($\varrho_1 \leqslant \varrho \leqslant \varrho_2$) where the heightened density causes the reinforcement of mutual interactions among vehicles. 3. The congested flow region ($\varrho > \varrho_2$) where the traffic is fully saturated and the movement of cars is therefore significantly restricted. Transitions among different traffic phases are schematically indicated by arrows. In the insert the schematic representation (main part of the figure) is compared to the empirical fundamental diagram.

It is obvious that mutual interactions among the agents cause the changes in the system dynamics, which finally results in relevant changes of macroscopic quantities for the system investigated (see [9] and [7]). Furthermore, macroscopic relations describing the global behavior of transport systems influence significantly the microscopic structure of the system. Such a structure is, as understandable, of statistical nature, which is caused by the individuality of each agent. Whereas for free flow states one can detect the random distribution of the system elements, for congested states the strong psychological repulsion among crowding agents leads to the strong systemization of the ensemble (see [13], [8]). Recently, these microscopic phenomenons are measurable ([1],[16],[15]), which opens new possibilities to inspect a local behavior in animal/human groups.

The main goal of this lecture is to obtain meaningful predictions for the microstructure of traffic sample (similarly to [2] or [10]) and compare the results obtained to freeway measurements.

2 Formulation of socio-physical transport model

Based on principles of quantitative sociodynamics sumarized in the book [5] we introduce the one-dimensional space-continuous thermal-like model whose microstructure (analyzed in the associated steady state) is in a good correspondence with that measured among real vehicles.

2.1 General model based on principles of quantitative sociodynamics

Consider N identical particles (agents) on the unit sphere (see the figure 2)

$$S = \left\{ \vec{\xi} \in \mathbf{R}^3 : \|\vec{\xi}\|_{\mathsf{e}} = 1 \right\},$$

where the symbol $\|.\|_{\mathsf{e}}$ corresponds to the standard (euclidean) norm, i.e.

$$\|\vec{\xi}\|_{\mathsf{e}} = \sqrt{\xi_1^2 + \xi_2^2 + \xi_3^2}.$$

Let $\vec{x}_i = (\vartheta_i, \varphi_i)$ $(i = 1 \dots N)$ denote the position of the ith particle, where ϑ_i and φ_i represent the spherical angles (latitude and longitude) of standard spherical coordinates. Let $\vec{v}_i$ stand for the actual velocity of ith particle and parameter $\vec{v}_{\mathrm{d}}$ is the desired velocity (the same for all agents). Denoting the general metric in the system as $\|\vec{x} - \vec{y}\|$ one can define the $\varepsilon-$neighborhood of the particle i according to

$$\mathcal{O}(\vec{x}_i) = \left\{\vec{\xi} \in \mathbf{R}^3 : \xi_1^2 + \xi_2^2 + \xi_3^2 = 1 \wedge 0 < \|\vec{\xi} - \vec{x}_i\| < \varepsilon\right\}.$$

Thus, the indexing set

$$I_i = \left\{k : \vec{x}_k \in \mathcal{O}(\vec{x}_i)\right\}$$

cumulates all the particles being inside the $\varepsilon-$neighborhood of the ith particle. We remark that the general metric $\|\vec{x} - \vec{y}\|$ can be chosen arbitrary (with reference to the systems observed), but the most usual choice is the euclidean metric.

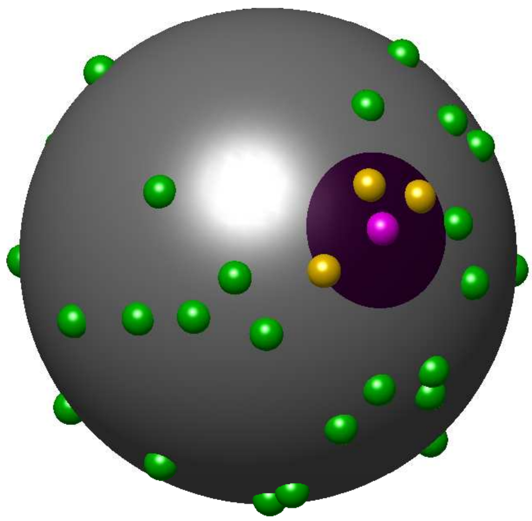

Figure 2: Agents on the Unit Sphere. The selected agent (magenta) and his/her $\varepsilon-$neighborhood. The magenta agent reacts only to the agents (yellow) occurring in his/her $\varepsilon-$neighborhood $\mathcal{O}$.

Aiming to quantify the mutual interactions among the agents we introduce the short-ranged potential energy

$$U \propto \sum_{i=1}^{N} \sum_{k \in I_i} V\left(r_{ik}\right),$$

where $V(r_i)$ corresponds to the repulsive two-body potential depending on the general distance $r_{ik} = \|\vec{x}_i - \vec{x}_k\|$ between the $\varepsilon-$neighboring particles only. The interaction of such a kind is chosen with the respect to the realistic behavior of animals/humans (see [13]). Besides, the potential $V(r)$ has to be defined so that $\lim_{r \to 0_+} V(r) = \infty$, which prevents particles from passing through each other. The socio-physical hamiltonian of the described ensemble reads as

$$\mathcal{H}_\varepsilon = \frac{m}{2}\sum_{i=1}^{N} \|\vec{v}_i - \vec{v}_{\mathrm{d}}\|_{\mathsf{e}}^2 + \mathsf{c}\sum_{i=1}^{N}\sum_{k\in I_i} V\left(r_{ik}\right),$$

where m represents a mass of particles and c is a calibrating coefficient. Whereas the second summand in the previous formula describes the particle attraction/repulsion the former takes into account the fact that driver moving with the desired velocity (being sufficiently far from other cars, where $V(r) \approx 0$) does not accelerate/decelerate.

2.2 Thermal alternative of the socio-physical transport model

The above-mentioned description is strictly deterministic and does not reflect statistical features of realistic human/animal communities. Therefore we introduce a thermodynamical alternative of the model (see [8] or [24]) where the entire system is exposed (if using the thermodynamical terminology) to a "heat bath" of a given temperature T, i.e. to random influences of a certain variance and statistics. Denote

$$\beta = (\mathrm{k_B}T)^{-1}$$

where $\mathrm{k_B}$ is the usual Boltzmann factor. The thermal parameter β can be interpreted as a psychological coefficient describing a level of the mental pressure under the driver is while driving his/her car. Hence, in the next part of this habilitation lecture we call β as a mental strain coefficient.

Implementing this thermal component into the originally deterministic system we have obtained the statistical ensemble whose thermal equilibrium is described statistically. It means that the microscopical quantities (gaps among the cars, velocities of single vehicles, time intervals among the subsequent cars and so on) measured in thermal equilibrium are determined by means of corresponding probability densities. This fact fully corresponds to the ascertainments observed during the traffic experiments (see [25] and [22]).

2.3 Circular variant of the model

Restricting the particle movement to the circular curve

$$C = \left\{\vec{\xi} \in \mathbf{R}^2 : \|\vec{\xi}\|_{\mathsf{e}} = 1\right\}$$

the previous thermal-like scheme converts to the simple model of one-lane traffic. In this case the location of each particle is unambiguously described by its angular coordinate φ_i. Then the hamiltonian of this one-dimensional system reads as

$$\mathcal{H} = \frac{m}{2}\sum_{i=1}^{N}(v_i - v_{\mathrm{d}})^2 + \mathsf{c}\sum_{i=1}^{N} V\left(r_i\right),$$

where

$$r_i := |\varphi_{i+1} - \varphi_i|\frac{N}{2\pi}$$

corresponds to the re-scaled circular distance between $(i+1)$th and ith particles. Denoting $\varphi_{N+1} = \varphi_1$ for convenience, the following equality holds true

$$\sum_{i=1}^{N} r_i = N. \tag{1}$$

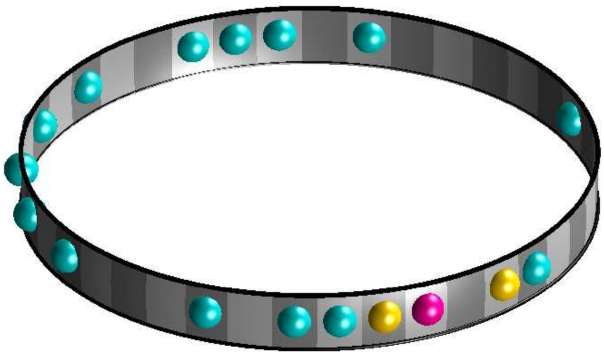

Figure 3: Vehicles on the Unit Circle. The interaction among the agents (drivers) is strictly short-ranged, which means that the particle (pink) interacts with two neighbors only (yellow). We remark that the circumference of the ring is equal to the number of particles. Thus, the mean distance between elements is equal to one.

As published in [14] the suitable choice for two-body traffic potential is the power-law function $V(r) = r^{-1}$. Under these conditions the corresponding partition function is of a form

$$\mathcal{Z} = \int_{\mathbf{R}^{2N}} \delta\left(N - \sum_{i=1}^{N} r_i\right) \prod_{i=1}^{N} \mathsf{e}^{-\frac{m}{2}\beta(v_i - v_{\mathrm{d}})^2} \prod_{i=1}^{N} \mathsf{e}^{-\frac{\beta}{r_i}} \mathsf{d}r_i \mathsf{d}v_i. \tag{2}$$

Here $\delta(x)$ stands for the generalized function called usually the Dirac $\delta-$function. After $2N-1$ integrations we find out that individual velocity v of particles is Gaussian distributed, i.e.

$$q(v) = \frac{1}{\sqrt{2\pi}\sigma}\,\mathsf{e}^{-\frac{(v-v_{\mathrm{d}})^2}{2\sigma^2}} \tag{3}$$

is the associated probability density, where $\sigma^{-1} = \sqrt{m\beta}$. Similarly, denoting by $\Theta(x)$ the Heaviside's step function

$$\Theta(x) = \begin{cases} 1, & x > 0 \\ 0, & x \leqslant 0 \end{cases}$$

and by $\mathcal{K}_\lambda(x)$ the modified Bessel's function of the second kind (the MacDonald's function) there has been derived in the article [14] that probability density for gap among the succeeding cars (clearance distribution) reads as

$$\wp(r) = A\,\Theta(r)\,\mathsf{e}^{-\frac{\beta}{r}}\mathsf{e}^{-Br}, \tag{4}$$

where

$$B = \beta + \frac{3 - \mathsf{e}^{-\sqrt{\beta}}}{2}, \tag{5}$$

$$A^{-1} = 2\sqrt{\frac{\beta}{B}}\mathcal{K}_1\big(2\sqrt{B\beta}\big). \tag{6}$$

We remark that $\wp(r)$ fulfils two normalization conditions

$$\int_{\mathbf{R}} \wp(r)\,\mathsf{d}r = 1 \tag{7}$$

and

$$\int_{\mathbf{R}} r\,\wp(r)\,\mathsf{d}r = 1. \tag{8}$$

The latter represents a scaling to the mean clearance equal to one.

3 Microscopic distributions in freeway-traffic samples

In this section we briefly summarize the methods for measurement and analysis of real-road traffic data. Furthermore, we will demonstrate that statistical properties of traffic micro-quantities (gaps among the cars, velocities of single vehicles, time intervals among the subsequent cars and so on) can be very well estimated by the steady-state distributions derived analytically for the above-mentioned thermodynamic traffic gas.

3.1 Traffic measurements

The first attempts to describe certain vehicular ensembles systematically, i.e. using mathematical techniques, were noticed more than seventy years ago. Indeed, in 1935 there was published the first scientific paper [4] on elementary relations among traffic quantities. Bruce Douglas Greenshields - author of this text - was probably the first man who carried out the traffic flow measurements (using photographic measurement techniques) and predicted the linear speed-density relation. Many mathematicians, physicists, or traffic engineers have followed up to his pioneering work since then. Recently, knowledge on behavior of traffic systems is very extensive (see reviews [17], [9], [7], or [11]) as well as the methods used for gaining individual traffic data.

Globally, the traffic measurement are divided into the following categories. *Intrusive methods* are mediated by measuring devices which are tightly inbuilt in a road surface. On contrary, *non-intrusive methods* are based on outlying measurements which are realized without any tight interconnection to a road surface. The intrusive methods include

- *induction-loop detector measurements* represented by the induction coil placed inside the road surface. The magnetic field of the induction loop is disturbed by the moving metallic bodywork of vehicle, which allows a registration of individual passages through the fixed point of freeway,
- *induction-double-loop detector measurements* represented by the two inter-connected induction coils placed inside the road surface. These double-loop detectors facilitate the detection of individual vehicle data, i.e. by mean of them we can analyze lengths of cars, individual velocities, time headways, or distance headways.

The non-intrusive methods include

- *photo-measurements* represented by terrestrial or aerial photography,
- *video-measurements* realized by terrestrial or aerial camera, or by floating cars,
- *toll-measurements* connected to the infrastructure of toll systems (toll gates),
- *ultrasonic detectors* based on principles of Doppler's radar,
- *microwave radars,*

- *laser scanning gauger.*

Majority of the devices mentioned above provides huge amounts of individual traffic data which can subsequently analyzed by the statistical or/and mathematical methods. Some of those techniques are introduced (and discussed) in the next part of this habilitation lecture.

3.2 Velocity distribution in freeway-traffic samples

Within the bounds of some older researches (published ten years ago) there have been studied the statistical distributions of individual velocities in detail. As published in the article [6] or summarized in the review [7] these distributions are depending on an actual traffic phase. It means that velocity distributions are different for free flows and congested flows. Recent investigations have substantially amplified this knowledge. In fact, statistical distributions of velocities are significantly influenced not only by the changes of phases but also by the slight changes of traffic densities. Indeed, analyzing data-samples from European highways we detect a marked dependence of function $q(v)$ on traffic density. Moreover, in all density intervals the probability densities for car velocities fully correspond to the analytical predictions (3) calibrated by the appropriate choice of parameters v_{d} and σ. Maximum likelihood estimation method (MLE) provides the relevant formulas for both parameters mentioned. It holds that $v_{\mathrm{d}} = \langle v \rangle$ and $\sigma^2 = m_2(v)$, where $\langle v \rangle$, $m_2(v)$ represent the sample mean and the sample variance, respectively. The chosen results of the corresponding analysis are presented in the figure 4.

3.3 Clearance distribution in freeway-traffic samples

Compared to the velocity distribution the distance clearance distribution has been systematically analyzed during the last ten years only. The attempts on meaningful predictions of distance statistics has been rare so far and are still a subject of speculations. However, in our articles [12] and [13] we have predicted the traffic microstructure (using the above-introduced method) with a marked success. Since then the thermal-like approach has been applied in another works ([14], [15], [1], [16], and [23]) and led finally to the partial solution of the traffic-clearances problem.

Considering now the tested choice for car-car potential $V(r) = r^{-1}$ we intend to compare the equilibrium distribution

$$\wp_\beta(r) = A\mathsf{e}^{-\frac{\beta}{r}}\mathsf{e}^{-Br} \tag{9}$$

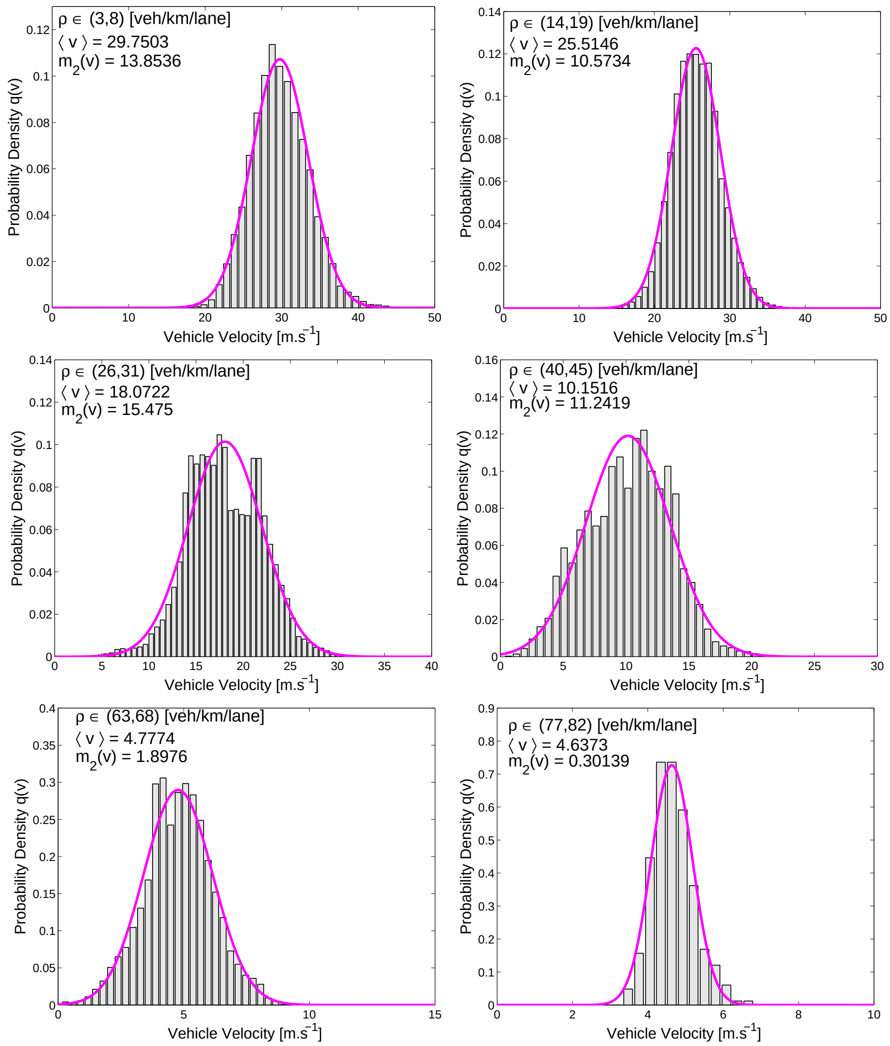

Figure 4: Vehicle Velocity Distribution. The bars represent velocities recorded on the Dutch freeway A9 using induction double-loop detectors. The entire data file has been divided into small density regions (see the texts inside the individual subplots) to separate the different traffic regimes. The continuous curves display the predictions (3) specified for $v_{\mathrm{d}} = \langle v \rangle$ and $\sigma = \sqrt{m_2(v)}$. Here $m_2(v)$ denotes the sample second central moment (i.e. sample variance).

with the relevant distributions of single-vehicle data measured continuously during approximately 140 days on the Dutch two-lane freeway A9. Macroscopic traffic density ϱ was calculated for samples of $N = 50$ subsequent cars passing a detector. For the purposes described above we divide the region of the measured densities $\varrho \in [0, 85\,\mathrm{veh/km/lane}]$ into 85 equidistant subintervals and separately analyze the data from each one of them. The sketched procedure prevents the undesired mixing of the states with the different inverse temperature β, i.e. with the different density. Bumper-to-bumper distance r_i among the succeeding cars (ith and $(i-1)$th) is calculated (after

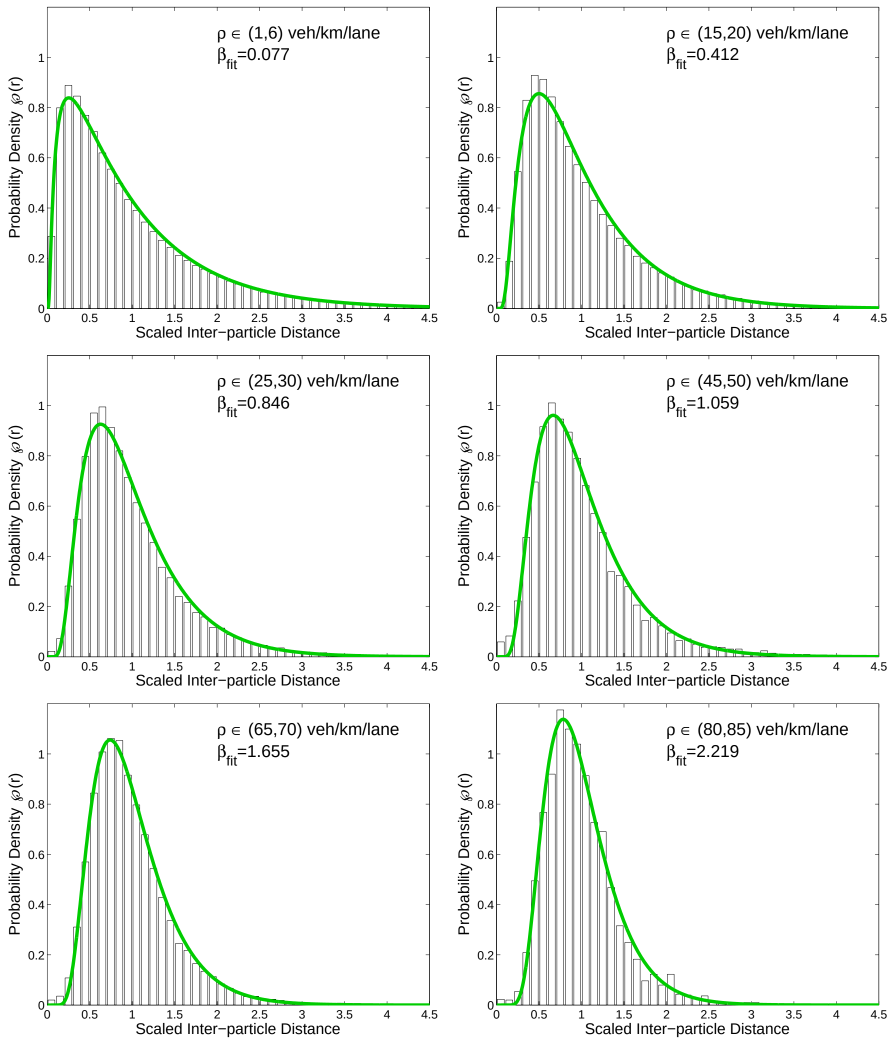

Figure 5: Distance Clearance Distribution. The bars represent the inter-particle gaps measured among the subsequent vehicles moving on a freeway. The entire data file has been divided into small density regions (see legend for details) to separate the different traffic regimes. The continuous curves display the predictions (4) for fitted value of inverse temperature β. Optimal value of β_{fit} was obtained by minimizing the χ^2-statistics.

eliminating car-truck, truck-car, and truck-truck gaps) via standard formula

$$r_i = v_i(t_i - t_{i-1}),$$

by means of netto time-headway $t_i - t_{i-1}$ and velocity v_i of ith car (both directly measured by induction-loop detectors) supposing that velocity v_i remains constant between the times t_i, t_{i-1} when ith car and the previous one are passing a measure point. Such a condition could be questionable, especially in the region of small densities where the temporal gaps are too large. However, the influence of a possible error is of marginal importance, as apparent from the fact, that cumulated distribution function plotted for small-density data does not show any visible deviation from exponential behavior of cumulated distribution function expected for independent events.

We note that mean distance among cars is re-scaled to one in all density-regions. The thorough statistical analysis of the traffic data leads afterwards to the excellent agreement between clearance distribution computed from traffic data and formula (9) for fitted value of inverse temperature β_{fit} (see the figure 5). We have obtained the fit parameter β_{fit} by a least-square method, i.e. minimizing the error function χ^2. The deviation χ^2 between the theoretical and empirical clearance distributions is plotted in the figure 6 (low part).

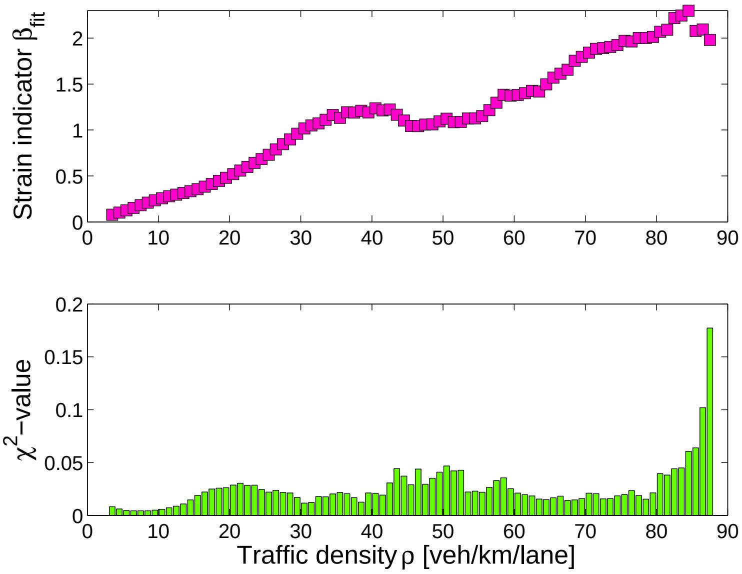

Figure 6: Inverse Temperature β_{fit} and Deviation χ^2 as a Function of Traffic Density ϱ. Squares stand for values of fit parameter β_{fit}, for which the function (4) coincides with clearance distribution of traffic data. Bars from lower part correspond to the sums of squared deviations between the empirical and the theoretical netto distance distributions for β_{fit}.

Dimensionless inverse temperature β of the traffic sample, representing a quantitative description of mental strain under which the car-drivers are in a given situation, shows non-trivial dependence on traffic density ϱ (as visible in the figure 6 – top part). For free flow states ($\varrho \lesssim 30$ veh/km/lane) one can recognize a rise in temperature having a linear behavior and visible plateau above. In the intermediate region (between 40 and 50 veh/km/lane), where free traffic converts to the congested traffic, we detect a sharp increase in the first half. Such a behavior can be simply elucidated by the fact that the drivers, moving quite fast in a relatively dense traffic flow, are under a substantial psychological pressure, which finally results (for densities $\varrho \in [40, 50]$ veh/km/lane) in the transition to the congested flows a therefore in the drop of inverse temperature. In the synchronized traffic regime ($\varrho \gtrsim$

50 veh/km/lane) the drivers vigilance rapidly grows up which culminates by the traffic-jam formation.

4 Summary and conclusions

Applying general methods of quantitative socio-dynamics we have presented the original one-dimensional thermal-like particle-ensemble whose time evolution (and the relevant steady state) is in deep consonance with time evolution investigated empirically in vehicular systems. Introducing the power-law repulsions among the neighboring particles of the model we have obtained a socio-physical alternative of the well-known Dyson's Coulomb gas that is powerful in prediction of statistical properties for ensembles of random matrices. Specifically, thermal-balance configurations of the particles in the Dyson's Coulomb gas are exactly the same as the set of random-matrix-eigenvalues (see [18]). Similarly, the above-mentioned power-law alternative of the Dyson's gas successfully predicts certain statistical properties of the traffic microstructure, especially distribution of individual velocities or distance/time headways/clearances. Whereas the prediction of Gaussian-distributed velocities extends the familiar piece of knowledge only, the predictions of clearance statistics have been a subject of speculations for years. However, implementing the socio-physical interactions (described by the power-law two-body potential $V(r) = r^{-1}$ depending on distances among the succeeding agents) into the original Dyson's model, the associated analytically-derived distributions for distance clearances show an excellent correspondence to traffic clearances measured on various European highways.

Moreover, the analyses of real-road traffic data-samples facilitate to inspect the behavior of vehicular ensembles in detail. Indeed, socio-physical description (used for derivation of the relevant predictions for traffic microstructure) allows to determine a level of psychological strains under which the drivers are if manoeuvring inside vehicular flows. Such a psychological feature of all traffic systems can be (according to our approach) quantified by the thermal-like indicator (here denoted β). As obvious from the empirical observations the mental strain indicator β is negligible for those traffic systems where density of vehicles is relatively small (compared to the critical density). On contrary, the psychological pressure is increasing with traffic density, which (after reaching the critical density) leads to the change of traffic phases (from free traffic regime to regime of congested states). Such a change is accompanied by the deep drop of individual speeds that causes therefore the temporal decrease of the strain indicator. If the number of cars is still expanding the indicator β is further increasing, which finally culminates by the creation of wide-spread traffic jam (the so-called stop-and-go phase).

To conclude, the presented thermal-like traffic model seems to be a suitable theoretical simulator of real-road vehicular streams since it produces the meaningful distributions of microscopic traffic quantities. The substantial deficiency of the approach presented is the fact that our model is of local substance and does not reproduce the macroscopical traffic effects (traffic hysteresis, congestions, and so on). Therefore, macroscopic alternatives of the thermal-like model are recently the topic of continuing researches.

References

[1] C. Appert-Rolland, Phys. Rev. E **80** (2009), 036102

[2] J. Baik, A. Borodin, P. Deift, and T. Suidan, J. Phys. A: Math. Gen. **39** (2006), 8965

[3] A.L. Barabási, Nature, London, **435** (2005), 207

[4] B.D. Greenshields, Proceedings of the Highway Research Board, Highway Research Board, Washington D.C., **14** (1935), 448

[5] D. Helbing, *Quantitative Sociodynamics: Stochastic Methods and Models of Social Interaction Processes*, Kluwer Academic, Dordrecht, (1995)

[6] D. Helbing, Phys. Rev. E **55** (1997), 3735

[7] D. Helbing, Rev. Mod. Phys. **73** (2001), 1067

[8] D. Helbing, M. Treiber, and A. Kesting, Physica A **363** (2006), 62

[9] D. Chowdhury, L. Santen, and A. Schadschneider, Physics Reports **329** (2000), 199

[10] D. Jezbera, D. Kordek, J. Kříž, P. Šeba, and P. Šroll, J. Stat. Mech. (2010), L01001

[11] B.S. Kerner, *The Physics of Traffic,* Berlin, Springer (2004)

[12] M. Krbálek, P. Šeba, and P. Wagner, Phys. Rev. E **64** (2001) 066119

[13] M. Krbálek and D. Helbing, Physica A **333** (2004), 370

[14] M. Krbálek, J. Phys. A: Math. Theor. **40** (2007), 5813

[15] M. Krbálek, J. Phys. A: Math. Theor. **41** (2008), 205004

[16] M. Krbálek and P. Šeba, J. Phys. A: Math. Theor. **42** (2009), 345001

[17] A.D. May, *Traffic Flow Fundamentals,* Prentice Hall, Englewood Cliffs, NJ (1990)

[18] M.L. Mehta, *Random matrices (revised and enlarged),* New York: Academic Press (1991)

[19] H. Moon, D.E. Conlon, S.E. Humphrey, N. Quigley, C.E. Devers, and J.M. Nowakowski, Organizational Behavior and Human Decision Processes **92** (2003), 67

[20] J.G. Oliveira, A.L. Barabási, Nature (London) **437** (2005), 1251

[21] J.G. Oliveira, A. Vazquez, Physica A **388** (2009), 187

[22] G. Orosz, R.E. Wilson, R. Szalai, G. Stépán, Phys. Rev. E **80** (2009), 046205

[23] D. Smith, J. Marklof, and R.E. Wilson, *Preprint* http://rose.bris.ac.uk/dspace/bitstream/ 1983/1061/1/dasmith.pdf, submitted to Eur. Phys. J. B (2008)

[24] A. Sopasakis, Physica A **342** (2004), 741

[25] Y. Sugiyama, M. Fukui, M. Kikuchi, K. Hasebe, A. Nakayama, K. Nishinari, S. Tadaki, and S. Yukawa, New Journal of Physics, **10** (2008), 033001

Mgr. Milan Krbálek, Ph.D.

Born:	13^{th} April 1971, Hlinsko, Czech Republic
Affiliation:	Department of Mathematics Faculty of Nuclear Sciences and Physical Engineering Czech Technical University, Prague, Czech Republic
Address:	Trojanova 13, 12000 Prague, Czech Republic Email: milan.krbalek⊛fjfi.cvut.cz Web pages: www.krbalek.cz Phone: (+420)-224-358-550

Education:

1994 – 1999	University of Hradec Králové (Master Degree)
1999 – 2003	Faculty of Nuclear Sciences and Physical Engineering (Doctoral Degree)

Scientific Fellowships:

1999 – 2003	Institute of Economy and Traffic, Technical University of Dresden, Germany
1999 – present	Max Planck Institute for Physics of Complex Systems, Dresden, Germany
2001 – present	Joint Institute of Nuclear Research, Dubna, Russia
1999 – 2003	Institute of Physics, Czech Academy of Science, Czech Republic
1999 – present	University of Hradec Králové, Czech Republic
2008 – present	Université Paris-Sud 11, Paris, France

Research Topics:

Spectral Analysis of Freeway Traffic Data
Analytical Computations for Physics of Traffic
Numerical Models of Traffic Flows
Asymmetric Simple Exclusion Model
Parking Cars Strategy (Analysis and Models)
Models for Pedestrian Flows
Models for Crowd Under the Panic Conditions
Deterministic Chaos in Classic and Quantum Physics
Random Matrix Theory

Citation Indexes:

ISI Web of Knowledge	**62** (with auto-citations)
ISI Web of Knowledge	**47** (without auto-citations)
H-index (ISI Web of Knowledge)	**05** (with auto-citations)

Scientific Collaborators:

Prof. RNDr. Petr Šeba, DrSc.	Random Matrix Theory, Theory of Chaos, Parking Problems, New Aspects in Physics of Traffic
Prof. Cecile Appert-Rolland	Microscopical Approaches in Physics of Traffic, Advanced Statistical Analysis of Traffic Data
Prof. Dr. Dirk Helbing	Quantitative Sociodynamics, Physics of Traffic, Local Thermodynamical Gases
Prof. Vyaceslav B. Priezzhev	Asymmetric Simple Exclusion Model
Prof. Ingrid Rotter	Classical and Quantum Chaos
Dr. Peter Wagner	Physics of Traffic, Cellular Models

List of Selected Publications:

2010

Milan Krbálek and Pavel Hrabák
Inter-particle gap distribution and spectral rigidity of totally asymmetric simple exclusion process with open boundaries
J. Phys. A: Math. Theor. (2010) – `submitted`

2010

Milan Krbálek
Discrete thermodynamical modelling of traffic streams
Selected Proceedings of the 12th World Conference on Transport Research Society, ISBN 978-989-96986-1-1
Lisbon, Portugal (2010)

2010

Milan Krbálek
Analytical derivation of time spectral rigidity for thermodynamic traffic gas
Kybernetika **46-6** (2010), 1108

2009

Milan Krbálek and Petr Šeba
Spectral rigidity of vehicular streams (Random Matrix Theory approach)
J. Phys. A: Math. Theor. **42** (2009), 345001

2008

Milan Krbálek
Inter-vehicle gap statistics on signal-controlled crossroads
J. Phys. A: Math. Theor. **41** (2008), 205004

2007

Milan Krbálek
Equilibrium distributions in a thermodynamical traffic gas
J. Phys. A: Math. Theor. **40** (2008), 5813

2005

Milan Krbálek
Dopravní systémy jako termodynamické plyny
Československý časopis pro fyziku **5** (2005), 432

2004

Milan Krbálek and Dirk Helbing
Determination of interaction potentials in freeway traffic

from steady-state statistics
Physica A **333** (2004), 370

2003 Milan Krbálek and Petr Šeba
Headway statistics of public transport in Mexican cities
J. Phys. A: Math. Gen. **36** (2003), L1

2001 Milan Krbálek, Petr Šeba, and Peter Wagner
Headways in the traffic flow
– remarks from a physical perspective
Phys. Rev. E **64** (2001), 066119

2000 Milan Krbálek and Petr Šeba
Statistical properties of the city transport in Cuernavaca (Mexico) and random matrix ensembles
J. Phys. A: Math. Gen. **33** (2000), L229

Textbooks:

2010 Milan Krbálek
Matematická analýza IV – cvičení
Česká technika - nakladatelství ČVUT, Praha 2010

2009 Milan Krbálek
Matematická analýza IV (druhé rozšířené vydání)
Česká technika - nakladatelství ČVUT, Praha 2009

2008 Milan Krbálek
Úlohy matematické fyziky
Česká technika - nakladatelství ČVUT, Praha 2008

2008 Milan Krbálek
Matematická analýza III (druhé rozšířené vydání)
Česká technika - nakladatelství ČVUT, Praha 2008

Popularizing Articles:

The Times (London), Discovery (USA), Science News, MF Dnes