České vysoké učení technické v Praze,
Fakulta stavební

Czech Technical University in Prague,
Faculty of Civil Engineering

RNDr. Vítězslav Vydra, CSc

Stanovení lomové energie

Determination of fracture energy of concrete

Habilitační přednáška

Abstract

It has been confirmed by many authors that specific fracture energy of concrete determined by laboratory experiments is dependent on the shape and size of the specimen because the local energy in the fracture proses zone is influenced by the free surface of the specimen. As the crack propagates towards the back face of the specimen shape of the fracture process zone changes which has a direct impact on local fracture energy. In this paper the author presents original concept of local fracture energy determination within a framework of effective crack model. Effective crack is a virtual notch of depth a_{e} in an ideally elastic sample which is identically deformed as a real sample with the plastic zone. Local fracture energy can be - within this model - determined using three-point-bend tests as a derivative of energy release rate with respect to effective crack length a_{e}. As long as the local fracture energy exhibits a plateau, *size-independent* fracture energy can be determined. The main advantage of suggested method of size-independent specific fracture energy determination is that it doesn't depend on testing of many samples of different shapes, sizes or notch to depth ratios.

Abstrakt

Mnoha autory bylo prokázáno, že specifická lomová energie betonu určená laboratorními testy závisí na tvaru a velikosti vzorků, protože lokální lomová energie je ovlivněna jejich volným povrchem. Při šíření trhliny směrem k opačnému konci vzorku se mění tvar lomové procesní zóny, což má přímý dopad na lokální lomovou energii. V tomto článku je představen originální způsob určování lokální lomové energie pomocí modelu efektivní trhliny. Efektivní trhlina (a_e) představuje myšlenou hloubku zářezu v dokonale elastickém vzorku, jehož tuhost je stejná jako tuhost reálného vzorku s plastickou zónou. Lokální lomová energie může být v rámci tohoto modelu určena pomocí tříbodového ohybu vzorků jako derivace rychlosti uvolňování deformační energie podle délky efektivní trhliny a_e. Pokud lomová energie vynešená jako funkce efektivní trhliny vykazuje široké maximum (plató), lze z hodnoty tohoto plató určit lomovou energii nezávislou na tvaru vzorku. Hlavní výhodou tohoto postupu je, že k určení lomové energie nezávislé na tvaru a velikosti vzorků není zapotřebí provádět testy na vzorcích různých velikostí resp. na vzorcích s různou hloubkou zářezu.

Keywords: local fracture energy, size effect, effective crack length

Klíčová slova: lokální lomová energie, efekt rozměru, délka efektivní trhliny

Contents

1 Introduction

Strength, stiffness, fracture toughness, specific fracture energy and brittleness are fundamental fracture properties of concrete. Strength and stiffness are directly used for the design and analysis of concrete structures under varied loading and environmental conditions. Toughness, specific fracture energy and brittleness are mainly used for the development of high-strength, high-performance concrete. Attempts have been made on using these properties for the design and analysis of concrete structures. Toughness and fracture energy commonly characterizes the capacity of a material to resist deformation and fracture. Brittleness is commonly understood to be the tendency for a structure to fracture abruptly before significant irreversible deformation occurs. Brittleness depends not only on the materials but also on the *size* of components. This means that design against brittleness is not only a pure material design but also an integrated material and structure design. The advent of high-strength concrete has led to new, cheap, high-performance structures, but in many cases led to big disappointments because the concrete has proved to be extremely brittle and unusable for load-carrying structures. Design of such structures has also gone beyond the limits of the existing concrete code even though the concept of brittleness has also been used to design the new, ultra-strong, cement-based materials and assess failure mode of the components made of these materials. The research on the concrete toughness and brittleness indeed helps further understand cracking and failure of conventional concrete materials and structures, helps to develop new extremely strong and ductile materials and achieve better and more logical and holistic design. So far, the information about the concrete toughness and brittleness is very limited and few parameters are used to assess this property. The main problem with most parameters is that they are size-dependent and parametres determined on small samples do not describe behaviour of large structures properly. Therefore, one of the hottest topics in the field of fracture mechanics of quasibrittle materials remains determination of corresponding size-independent material properties applicable for structures of any size.

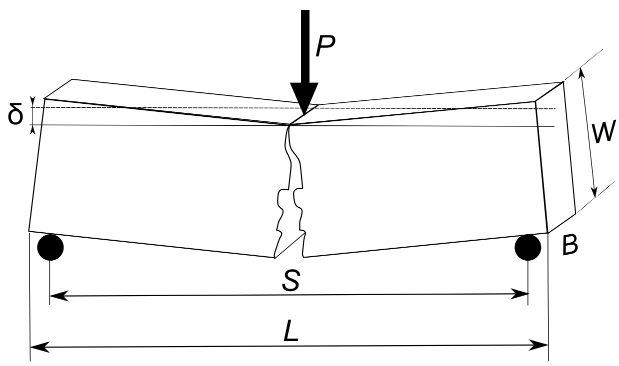


Figure 1: Three-point-bending test arrangement. δ is displacement of the loading point during bending (deflection of the beam).

The specific fracture energy G_F determined from three-point-bend tests is probably the most frequently used parameter in fracture analysis of concrete structures. The fracture energy G_F according to the Rilem recommendation [10] is average specific energy obtained as a ratio of total work of fracture and the fracture area (i.e. cross-section of initially uncracked ligament).

For a specimen of depth W, thickness B and initial notch depth a_0 (see fig. 1), the fracture energy is given by

$$G_\mathrm{F} = \frac{\int_0^{\delta_\mathrm{max}} P d\delta + \frac{1}{2} m g \delta_\mathrm{max}}{(W - a_0)\, B}, \tag{1}$$

where P is applied load, δ is displacement of the load point (see Fig. 5 for illustration of the load-displacement curve), δ_{max} is displacement of the load point at which applied load reaches zero, m is mass of the specimen and g is acceleration due to gravity. It is well known, however, that G_F varies with sizes and shapes of test specimens. To avoid this variability in the specific fracture energy determination Rilem committee issued guidelines for specimen sizes and test arrangement. The main drawback of the Rilem recommendations, however, are quite *enormous sizes* of specimens.

Many researchers have tried to identify reasons for variability of G_F with sizes of specimens. Hu and Wittman [5] have addressed the possibility that specific fracture energy may vary along the crack path as it propagates throughout a relatively thin sample. Such *local* energy being a function of crack length a is denoted as $g_f(a)$ to distinguish it from the averaged G_F (see Eq. 4). Model of Duan et al. [2] which addresses this problem is based on an assumption that energy required for the crack propagation depends also on the fracture-process-zone (FPZ) width w. The FPZ width is affected by free boundary of the specimen and therefore local fracture energy g_f is a function of position of the crack tip in the specimen. Hu and Wittman [6] suggested bi-linear fracture energy distribution along the crack tip path (see Fig. 2).

$$g_f(a) = \begin{cases} G_\infty & a < W - a_l \\ G_\infty \left(1 - \frac{a-(W-a_l)}{a_l}\right) & a \geq W - a_l, \end{cases} \tag{2}$$

where G_∞ is size independent fracture energy and a_l is ligament transition length.

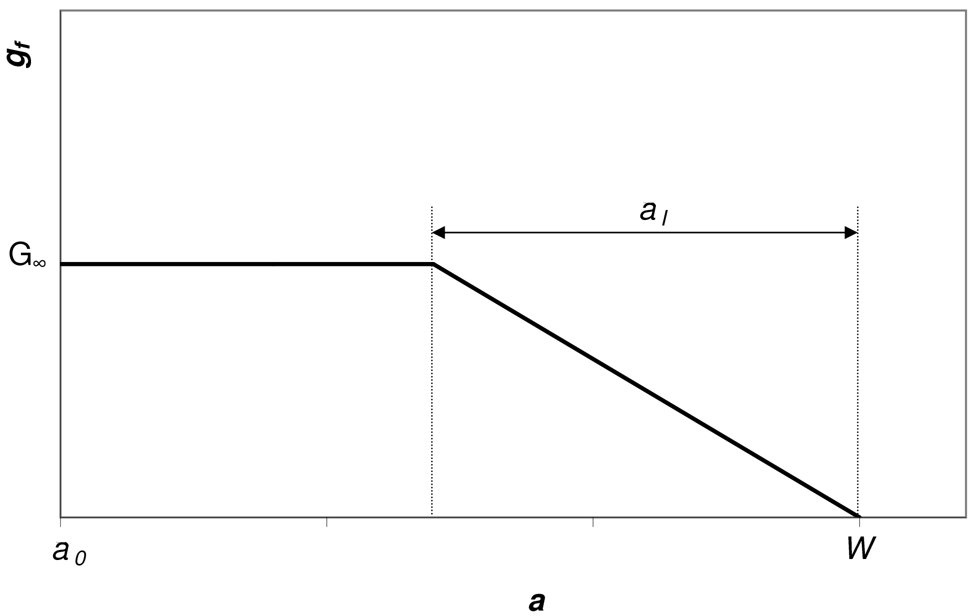


Figure 2: Bi-linear model of $g_f(a)$ [6]

Substituting Eq. 2 into Eq. 4 they got

$$G_F\left(\frac{a_0}{W}\right) = \begin{cases} G_\infty \left(1 - \frac{a_l}{2W(W-a_0)}\right) & W - a_0 > a_l \\ G_\infty \left(1 - \frac{(W-a_0)}{2a_l}\right) & W - a_0 \leq a_l. \end{cases} \tag{3}$$

Fracture energy G_F can be then determined using local energy concept as [3, 2]

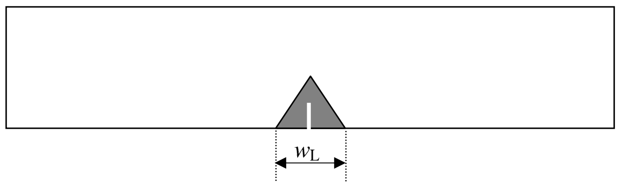
$$G_F = \frac{1}{(W-a_0)\,B} \int_{a_0}^{W} g_f(a)\, da. \tag{4}$$

Two similar methods based on measurements of G_F on samples with several (minimum two for bi-linear model) notch depth has been suggested for $g_f(a)$ determination [1, 3]. These methods assume that a notch with a depth a_0 is equivalent to a crack with the same length. Model of a crack equivalent elastically to a notch with depth a_e has already been published by Nallathambi and Karihaloo [8] as the well known effective crack model.

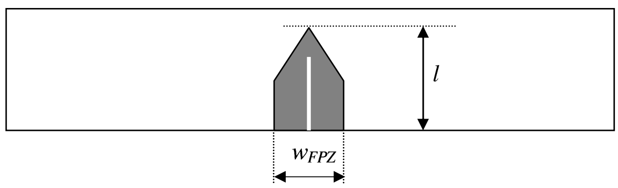
In this paper the concept of local fracture energy in viewpoint of the effective crack model is further developed. Local fracture energy is by the author of this paper determined as a derivative of energy release rate with respect to effective crack length a_e using entire records of load-displacement curves of three-point-bending tests of concrete beams. The size-independent fracture energy (denoted as G_∞) can be determined by this method as long as the local fracture energy exhibits a plateau.

2 Local fracture energy determination within effective crack model

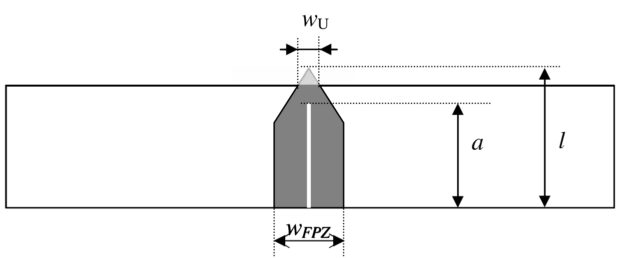
Rilem fracture energy concept was originally developed by Hilleborg [4] and was based on Hilleborg's fictitious crack model which ignores fracture process zone width. Local fracture energy concept [5] expands this model as it accepts that fracture process zone width w varies as the crack propagates through the specimen (as it is demonstrated schematically by author of this paper on Fig. 3) and consequently accepts that



(a) Initial shape of the fracture process zone is triangular.



(b) Width w_{FPZ} is constant as soon as the fracture process zone is fully developed.



(c) Fracture process zone reaches the specimen backface finally.

Figure 3: Fracture process zone shape changes as the fracture propagates throughout the sample and therefore energy required to its propagation changes (schematically - bending of the beam is not shown).

experimentally determined value of G_F changes with the specimen size and notch depth.

In the first approximation it is possible to suppose that energy $\mathfrak{g}$ required for creating of a unit volume of the process zone is *constant*. Local energy g_{f} can be than expressed as a function of fracture process zone volume V_{FPZ} as

$$g_{\mathrm{f}} = \frac{\mathfrak{g}}{B}\frac{dV_{\mathrm{FPZ}}}{da_{\mathrm{e}}}.$$

Although the effective crack length a_{e} was originally suggested to be evaluated at ultimate load P_{u} only [8] the same calculation procedure can be easily applied to any value of load and displacement – see Fig. 5. For this purpose the author of this paper developed a new straightforward method of effective crack length determination which is described in section 4. Fracture process zone volume V_{FPZ} can be in first approximation according the Fig. 3 expressed as:

$$\frac{dV_{\mathrm{FPZ}}}{da_{\mathrm{e}}} = \begin{cases} B\; w_{\mathrm{L}}(a_{\mathrm{e}}) & w_{\mathrm{L}} < w_{\mathrm{FPZ}} \quad \text{(a)} \\ B\; w_{\mathrm{FPZ}} & w_{\mathrm{L}} > w_{\mathrm{FPZ}} \text{ and } l < W \quad \text{(b)} \\ B\,(w_{\mathrm{FPZ}} - w_{\mathrm{U}}(a_{\mathrm{e}})) & l > W \quad \text{(c)}, \end{cases} \tag{5}$$

where equations (a) – (c) are related to sub-figures (a) – (c) on Fig. 3 respectively, w_{L} and w_{U} is fracture process zone width on lower (front-face) and upper (back-face) surfaces of a beam, w_{FPZ} is width of fully developed fracture process zone. This equations can be further simplified on assumption that $dw/da_{\mathrm{e}} = const.$:

$$\frac{dV_{\mathrm{FPZ}}}{da_{\mathrm{e}}} = \begin{cases} B\left(\frac{dw_{\mathrm{L}}}{da_{\mathrm{e}}}a_{\mathrm{e}} + w_{01}\right) & \text{(a)} \\ B\, w_{\mathrm{FPZ}} & \text{(b)} \\ B\left(w_{02} - \frac{dw_{\mathrm{U}}}{da_{\mathrm{e}}}a_{\mathrm{e}}\right) & \text{(c)}, \end{cases} \tag{6}$$

where derivatives $dw_{\mathrm{L}}/da_{\mathrm{e}}$, $dw_{\mathrm{U}}/da_{\mathrm{e}}$ and also w_{01} and w_{02} are constant.

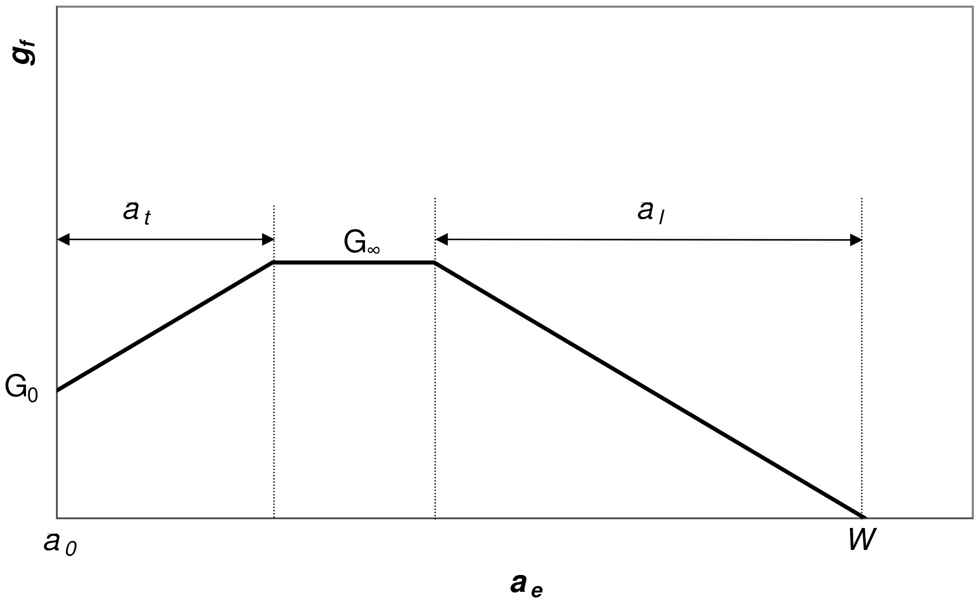


Figure 4: The distribution of fracture energy g_{f} along the notched beam of depth W and notch depth a_0.

Finally (introducing another symbols meaning of which is obvious from Fig. 4) fracture energy g_f can be expressed as a *trilinear* function of effective crack length as:

$$g_\mathrm{f}\left(a_\mathrm{e}\right) = \begin{cases} \frac{G_\infty - G_0}{a_\mathrm{t}}\left(a_\mathrm{e} - a_0\right) + G_0 & a_\mathrm{e} < a_\mathrm{t} + a_0 \\ G_\infty & a_\mathrm{t} + a_0 < a_\mathrm{e} < W - a_\mathrm{l} \\ G_\infty - \left(a_\mathrm{e} - W + a_\mathrm{l}\right)\frac{G_\infty}{a_\mathrm{l}} & a_\mathrm{e} > W - a_\mathrm{l} \end{cases} \tag{7}$$

3 Experimental determination of the local fracture energy

Crack development is always driven by energy released by the specimen. The so called energy release rate $\mathcal{G}$ can be determined by differentiation of potential energy of external and elastic forces with respect to effective crack length:

$$\mathcal{G} = -\frac{1}{B}\frac{d\Pi}{da_\mathrm{e}} = -\frac{1}{B}\left(\frac{d(\Pi_e + \Pi_\mathrm{P})}{da_\mathrm{e}}\right). \tag{8}$$

$\Pi_\mathrm{P} = -\int_0^\delta P(x)dx$ is potential energy of external forces and $\Pi_\mathrm{e} = \frac{1}{2}P(\delta)\cdot\delta$ is potential energy of elastic forces. Under quasi-static conditions of loading the local fracture energy g_f is equal to energy release rate:

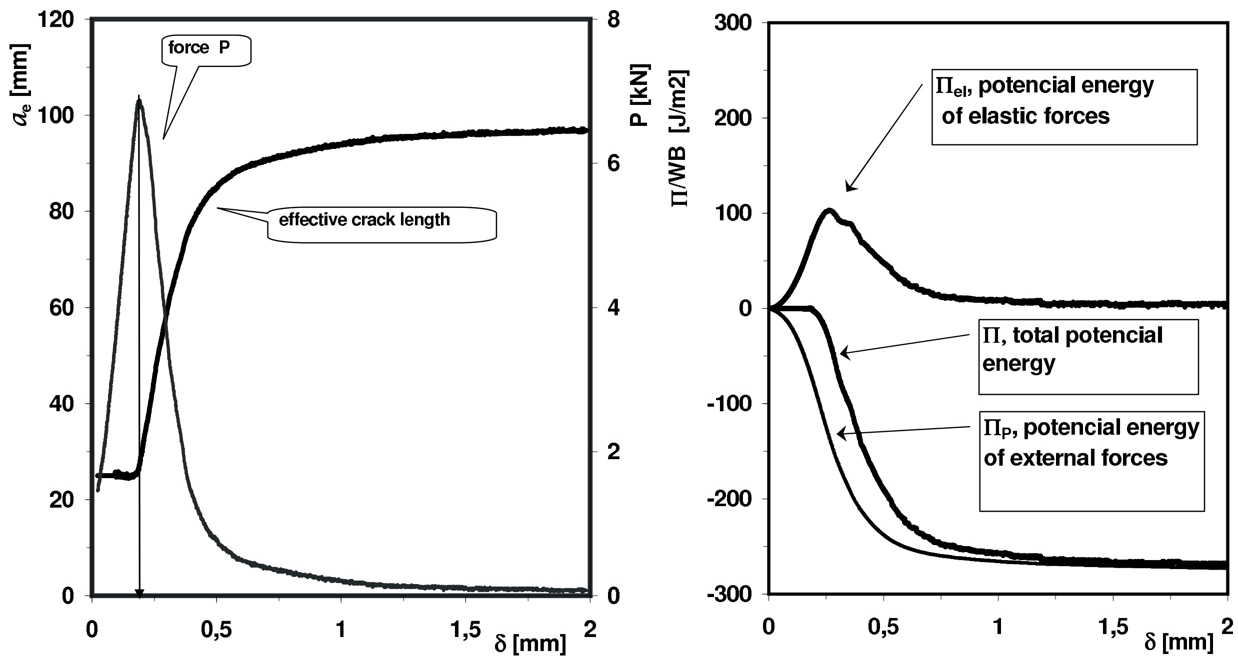


Figure 5: Load-displacement curve and the effective crack length a_e determined by Eq. 12 for arbitrary value of a load point displacement δ (left). Potential energy of external and elastic forces Π as a function of displacement of the load point δ. Normalized to initial ligament cross-section $B(W - a_0)$ (right).

$$g_\mathrm{f}(a_\mathrm{e}) = \frac{1}{B}\frac{d\left(\int_0^\delta P(x)dx - \frac{1}{2}P(\delta)\cdot\delta\right)}{da_\mathrm{e}}. \tag{9}$$

Eq. 9 can be solved using two different methods:

1. Integral in Eq. 9 is evaluated numerically and than derivation with respect to a_e is performed (numerically, too).

2. Integral in the Eq. 9 is performed numerically and fitted by any suitable mathematical function of a_e using the least square method. The fitted curve is derived with respect to a_e analytically. This method should be preferred as it allows to avoid cumbersome numerical derivation. As a suitable expression for fitting may be chosen integrated Eq. 7 i.e. $\Pi(a_e) = \int_0^{a_e} g_f(a)\, da$:

$$\Pi(a_e) = \begin{cases} \frac{G_\infty - G_0}{2a_t}(a_e - a_0)^2 + G_0 a_e + c_1 & a_e < a_t + a_0 \\ G_\infty a_e + c_2 & a_t + a_0 < a_e < W - a_l \\ G_\infty a_e - (a_e - W + a_l)^2 \frac{G_\infty}{2a_l} + c_3 & a_e > W - a_l, \end{cases} \tag{10}$$

where constants c_1, c_2 and c_3 should ensure that curves are connected. Example of fitting of this kind as well as fitting using polynomial function is presented on Fig. 6.

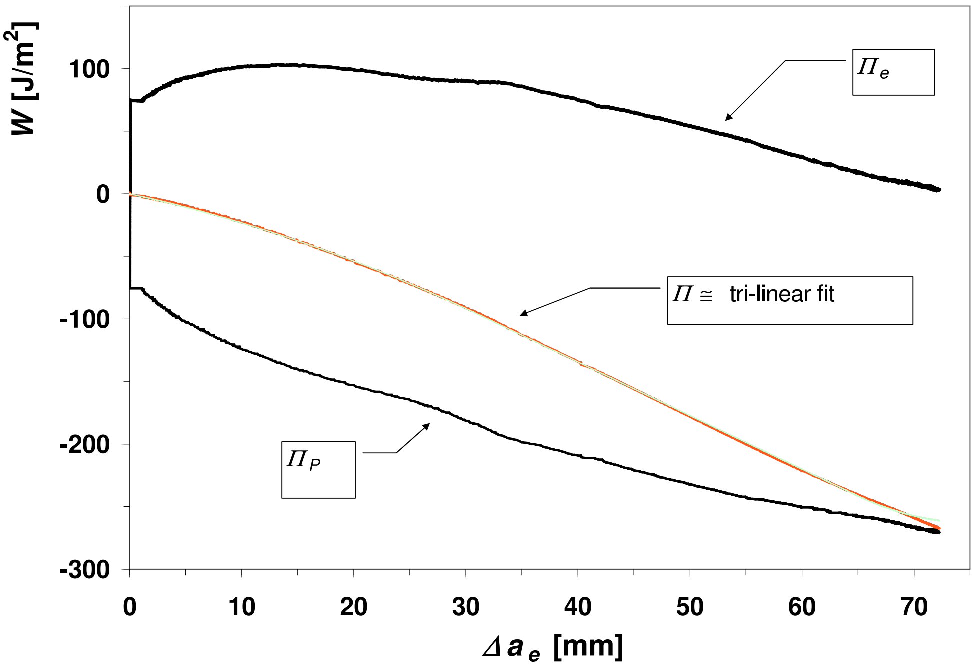


Figure 6: Potential energy of external and elastic forces Π fitted by tri-linear function 10 as a function of effective crack prolongation $\Delta a_e = a_e - a_0$.

Results of all above mentioned methods are presented on Fig. 7. It can be seen that different methods give similar results exhibiting reasonable plateau between ascending and descending branches which can be interpreted as an experimental prove of Eq. 7. If there were no plateau observable the ligament width would not be sufficient for size-independent fracture energy G_∞ determination.

As soon as effective crack reaches the specimen back-face ($\Delta a_e \to W - a_0$), polynomial as well as direct derivative methods fail. Reason for this is obvious: accuracy of determination of effective crack length is much worse in this region.

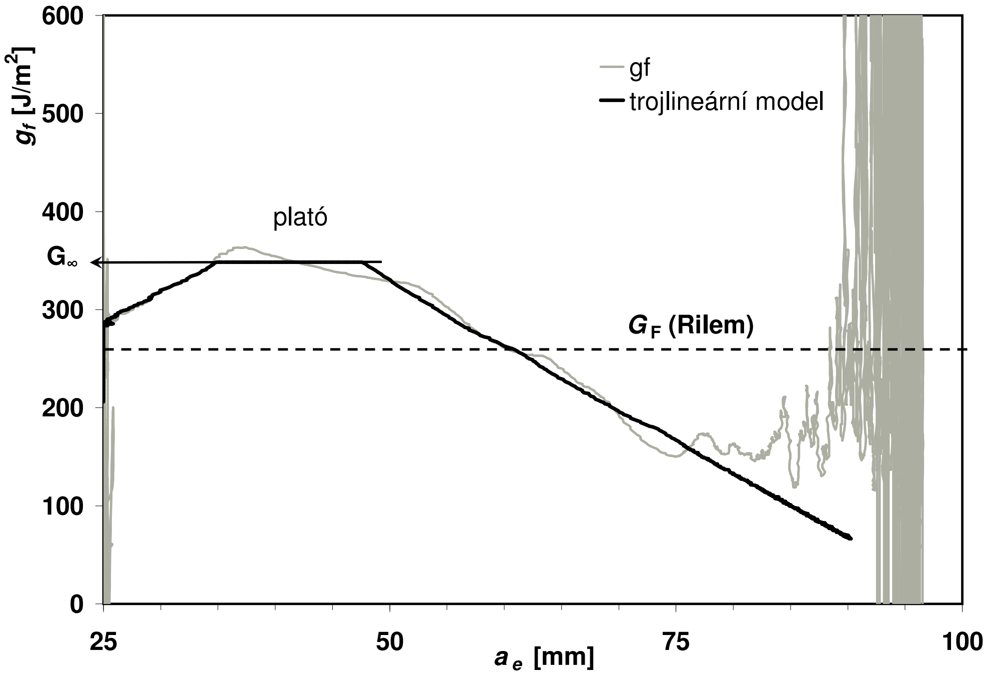


Figure 7: Local specific fracture energy g_f determined by two different numerical methods. The plateau height is the size independent fracture energy G_∞.

4 Determination of the effective crack length

Several different methods of effective crack length determination has been published [7, 11]. Implicit equation of Stibor was proved by FEM studies as the most accurate:

$$E = \frac{1}{B}\left\{\frac{P}{4\delta}\left(\frac{S}{W}\right)^{3}\left[1-0.387\frac{W}{S}+12.13\left(\frac{W}{S}\right)^{2{,}5}\right]+\frac{9}{2}\frac{P}{\delta}\left(\frac{S}{W}\right)^{2}F_1\left(\alpha_\mathrm{e}\right)\right\}, \qquad (11)$$

where $\alpha_\mathrm{e} = a_\mathrm{e}/W$ is relative crack length, S is span of the beam, E is modulus of elasticity, $F_1\left(\alpha_\mathrm{e}\right) = \int_0^{\alpha_\mathrm{e}} xY^2\left(x\right)dx$, and Y is function of geometry according to Pastor [9], which is suitable for any sample geometry:

$$Y\left(\alpha\right) = Y_\infty\left(\alpha\right) + 4\frac{W}{S}\left(Y_4\left(\alpha\right) - Y_\infty\left(\alpha\right)\right),$$

where

$$Y_4\left(\alpha\right) = \frac{1.9-\alpha\left[-0.089+0.603\left(1-\alpha\right)-0.441\left(1-\alpha\right)^2+1.223\left(1-\alpha\right)^3\right]}{\left(1+2\alpha\right)\left(1-\alpha\right)^{3/2}}$$

and

$$Y_\infty\left(\alpha\right) = \frac{1.989-\alpha\left(1-\alpha\right)\left[0.448-0.458\left(1-\alpha\right)+1.226\left(1-\alpha\right)^2\right]}{\left(1+2\alpha\right)\left(1-\alpha\right)^{3/2}}.$$

Relative crack length α_e is normally evaluated numerically as a root of implicit Eq. 11, but it can be expressed approximately in explicit way, too

$$
\begin{aligned}
\alpha_e(F_1) &= \frac{1}{2} + \frac{\arctan(b_1 + b_2 \ln F_1 + b_3(\ln F_1)^2 + b_4(\ln F_1)^3)}{\pi}, \text{ where} \\
b_1 &= 0.2872 + 0.2091\frac{W}{S}, \\
b_2 &= 0.5268 - 0.0067\frac{W}{S}, \\
b_3 &= 0.0381 + 0.017\frac{W}{S}, \\
b_4 &= 0.0284 - 0.002\frac{W}{S},
\end{aligned}
\tag{12}
$$

and the function

$$
F_1 = \frac{2B}{9}\left(\frac{W}{S}\right)^2 \left(\frac{E\delta}{P} - \frac{1}{4B}\left(\frac{S}{W}\right)^3 \left(1 - 0.387\frac{W}{S} + 12.13\left(\frac{W}{S}\right)^{2,5}\right)\right),
$$

has been determined using Eq. 11.

This formula is valid for width to span ratio W/S within an interval $(0.1, 1)$ with accuracy better than 0.7%. Modulus of elasticity E is obtained from three point bending tests by means of formula 11 using initial slope of *l-d* curves for P/δ ratio and initial notch depth to depth-of-specimen ratio (a_0/W) for α.

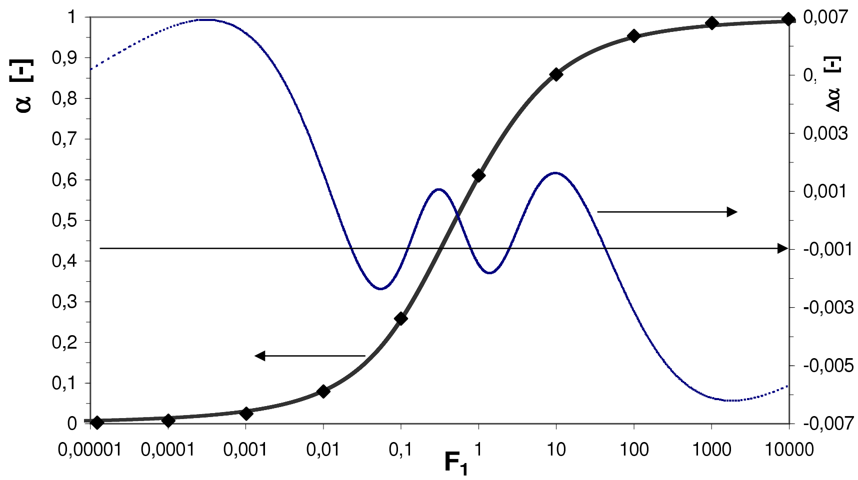


Figure 8: Function 12 and its error. Exact values are plotted as points and the function is the full line. Doted line is difference between exact value and the function 12.

5 Experimental

A set of ordinary concrete samples made of Portland cement was investigated by suggested method. Samples were exposed to elevated temperatures up to 1000°C in age of 180 days. Three point bend testing was carried out using close-loop testing machine. Three samples were used for every temperature. All samples were of uniform sizes i.e. B =100 mm, W =100 mm, beam span S = 300mm, length of samples L =400 mm and initial notch depth a_0 = 25 mm (see figure 1).

G_∞ experimentally determined using suggested method is presented on Fig. 10 and compared with fracture energy G_F calculated according to Rilem formula 1. Relative

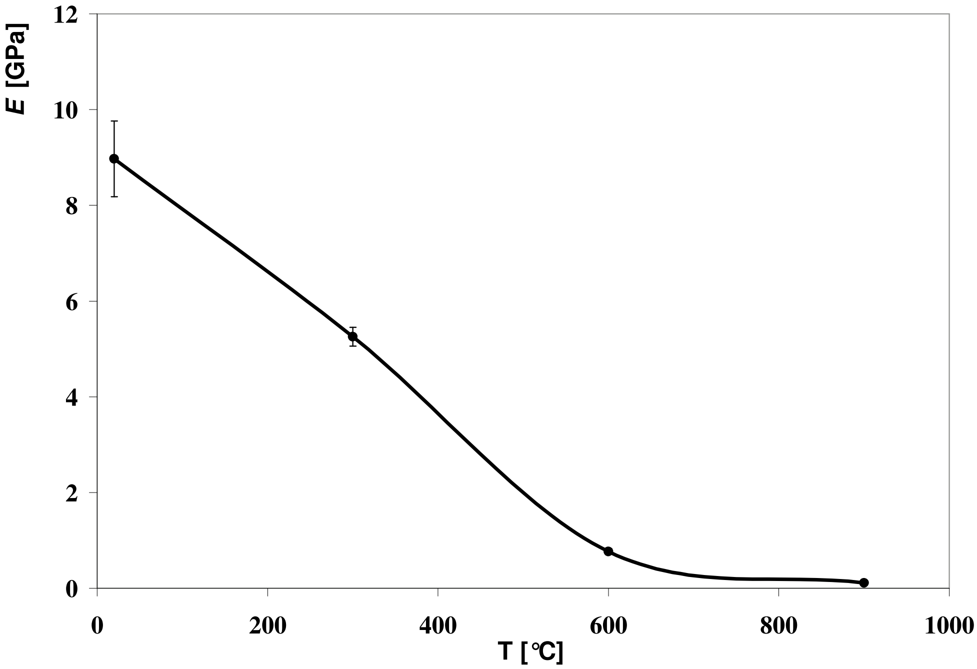


Figure 9: Modulus of elasticity obtained from three point bending tests by means of formula 11 when initial slope is substituted for P/δ and a_0/W for α. Error bars is 50% probable error Δ_{50}.

values of G_∞/G_F behave as expected i.e. remain in reasonably narrow range of values being always higher than one.

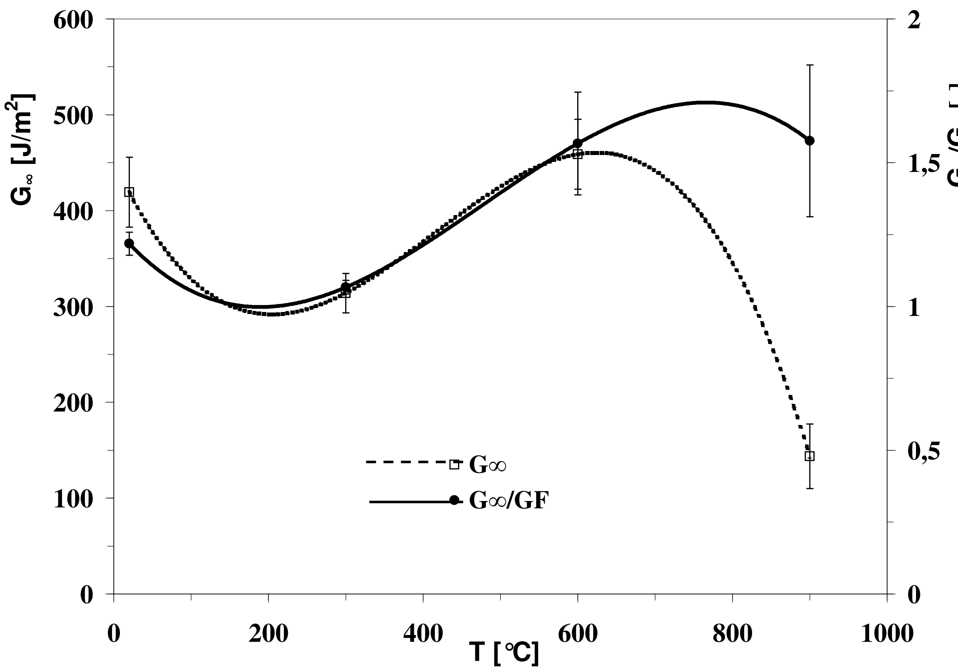


Figure 10: Size independent fracture energy G_∞ and its relation to G_F determined according to Rilem (Eq. 1).

Acknowledgments

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RNDr. Vítězslav Vydra, CSc

Education

- 1979–1984 study at Charles University in Prague, Faculty of Mathematics and Physics. Specialization: physics of solid state, magnetic properties of solids, ferromagnetic resonance of yttrium garnets.
- 1984 Doctor of natural sciences
- 1989 Ph.D., Physical Institute of Czechoslovak Academy of Sciences

Employment

- 1984–1989 Physical Institute of Czechoslovak Academy of Sciences
- since1989 employed at Faculty of civil Engineering, CTU Prague

Research

- 1993–1996 Member of the research team resolving the project "Management of aging of concrete containment buildings" at International Atomic Energy Agency.
- 1999–2004 Member of the research team resolving the project "Nuclear facilities, nuclear safety and radiation protection", No. MSM 210000020 at Ministry of Youth and Education.
- 2000–2002 Member of the research team resolving the project "Enhancement of national ability of assessment of concrete structures used in nuclear power plants", research project No. ME 378.
- 2003–2005 Member of the research team resolving the project "Research of early stages of hydration of the cement paste", grant No. GA 106/03/0028 of The Czech Science Foundation.
- since 2000 Member of the research team resolving the project "Safety of Nuclear Power Plants", research project No. MSM 6840770020 at Ministry of Youth and Education.

Teaching

- basic courses of physics
- physical topics in environmental problems
- transport of heat and moist in structures