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Aplikace systémů se zpožděním na řízení objektů s rozloženými parametry

Application of time delay system theory to control design of plants with distributed parameters

Annotation

Application of linear time delay system models for modeling and control of plants with distributed parameters is presented. First, the structure of the state space model of time delay system is designed, which is given by a set of functional differential equations. As an example of the time delay based modeling approach, a model of a laboratory heating system is presented. Even though the system consists of two heating circuits with heater, heat exchanger and cooler connected by pipelines, its model consists of only four differential equations with time delays. Let us remark that the state variables of the model are the temperatures, which are considered as the model outputs and which are measured on the heating system. Such a unification of the state and output variables in modeling the systems with distributed parameters is one of the main benefits of time delay based modeling approach. On the other hand, it is necessary to emphasize that due to functional character of the model, its dynamics analysis and control design requires application of advanced numerical and analytical design methods. One of them is the algorithm for computing large parts of the spectrum of time delay model which is also explained in this text. The algorithm is based on mapping the quasi-polynomial characteristic function using a numerical mapping method combined with analytical approaches for determining the asymptotic features of the spectrum distribution. As an example of a control design method, an extension of the synthesis of the well known state feedback control towards the systems with time delays is presented. Besides the theoretical issues, also implementation of state feedback control to the heating system is presented. The last topic presented is the synthesis of a controller based on internal model control platform with the first order model with two time delays as the reference model. Since such a model can be used to describe the dynamics of large scale of real systems, which are conventionally described by higher order models, the resulting control algorithm can be applied in a large scale of engineering applications. Finally, the control algorithm is discretized, implemented on a programmable controller and successfully tested on the laboratory heating system.

Anotace

Habilitační přednáška se zabývá využitím lineárních matematických modelů s dopravním zpožděním k popisu a následně k syntéze řízení systémů s rozloženými parametry. Nejprve je definován stavový popis modelu s dopravním zpožděním, který je dán soustavou funkcionálních diferenciálních rovnic. Následně je uveden příklad modelu laboratorní tepelné soustavy, která se skládá ze dvou tepelných okruhů. I přesto, že se jedná o poměrně složitý systém, jehož hlavními částmi jsou ohřívač, tepelný výměník a chladič, matematický model popisující dynamiku tohoto systému v okolí zvoleného pracovního bodu se skládá pouze ze čtyř diferenciálních rovnic se zpožděními, jehož stavovými veličinami jsou teploty měřené na soustavě. Právě možnost unifikace stavových veličin s výstupními veličinami modelu při popisu systémů s rozloženými parametry je významnou výhodou této třídy modelů. Na druhou stranu je nutné zdůraznit, že vzhledem k funkcionální povaze modelů se zpožděními, vyžaduje analýza jejich dynamiky a syntéza řízení použití pokročilých analytických či numerických metod. Jednou z těchto metod je algoritmus pro výpočet rozsáhlých částí spekter modelů se zpožděními, který je též popsán v tomto textu. Algoritmus spočívá v mapování kvazi-polynomiální charakteristické funkce systému pomocí numerické mapovací metody s využitím analytických metod pro určení asymptotických vlastností spektra. Jedním z klasických algoritmů řízení je tak zvaný stavový regulátor, jehož syntéza modifikovaná pro systémy se zpožděním je též jedním z témat habilitační přednášky. Kromě teoretického rozboru syntézy je uveden příklad aplikace stavového regulátoru na řízení laboratorní tepelné soustavy. Posledním prezentovaným tématem je syntéza regulátoru s vnitřním modelem prvního řádu se dvěma dopravními zpožděními. Vzhledem k tomu, že zmíněný model lze použít pro aproximaci dynamiky široké třídy systémů obvykle popisovaných modely vyšších řádů, má výsledný řídicí algoritmus potenciál širokého uplatnění v inženýrských aplikacích. Tento řídicí algoritmus je dále převeden do diskrétního tvaru, implementován na bázi programovatelného automatu a úspěšně testován na laboratorní tepelné soustavě.

Klíčová slova: teorie řízení, model dynamiky systému, systém s dopravním zpožděním, identifikace parametrů modelu, spektrum systému, stavový regulátor, řízení s vnitřním modelem

Keywords: control theory, model of system dynamics, time delay system, parameter identification, system spectrum, state feedback control, internal model control

CONTENTS

1. INTRODUCTION

Time delay systems (TDS), as a subset of hereditary systems, are largely used to describe the transport and propagation phenomena, time-lags, after-effects or systems with distributed parameters. The applications can be found in heat transfer, chemical and combustion processes, distributed networks, congestion control (traffic control, internet modeling) or even economics or biology (population dynamics modeling), see e.g. (Hale and Verduyn Lunel 1993), (Kolmanovskii and Myshkis, 1992).

At the Department of Instrumentation and Control Engineering and the Centre of Applied Cybernetics, two laboratory heating systems have been built in order to test the TDS based modelling techniques and control design methods. The first (older) system will be described in section 2.1. The second system, designed by the author, is shown in Fig. 1. It consists of two independent heating circuits equipped with heaters, coolers, pumps and mixing valves connected by pipelines. The heat exchange between the circuits takes place in the multi-plate heat exchanger. The system is monitored by twelve thermometers. Temperature measurement processing and system control can be done either via PC equipped with data acquisition cards or via PLC equipped with analog input-output modules. Naturally, time delays arise as the consequence of the transportation phenomenon in the long pipelines. However, using the delays also in modelling of the components with the capacities and distributed parameters (heaters, coolers, exchanger) allows us to build models of such subsystems which are more natural than delay free models.

Basically, using not only integrators but also delay blocks as the dynamical elements of the model gives us more power in building models of real plants. On the other hand, the dynamics of TDS is infinite dimensional, which requires to extend the theoretical tools for system design, e.g., from the usual rational transfer functions to meromorphic (or transcendental) ones.

The aim of this text is to demonstrate the potentials of application of TDS theory in the engineering applications, namely in the control design of systems with distributed parameters. First, in Section 2, the structure of linear TDS model is introduced and the model of a laboratory heating system is used as an application example of TDS model. Then, the features of the TDS dynamics are briefly outlined and the algorithm for computing large parts of the spectrum of TDS is introduced in Section 3. The algorithm is then utilized in direct and continuous pole placement methods in Sections 4 and 5, respectively, which are presented as examples of TDS control design methods. Also a real plant application example (in Section 4) and theoretical example (in Section 5) are presented. In Section 6 first order TDS model with two delays is introduced as a universal model unit. Based on this model an anisochronic controller is derived using an internal model control scheme. Implementation of the controller in an industrial PLC and its application to the laboratory heating system is included. Finally, in Sections 7 concluding remarks can be found.

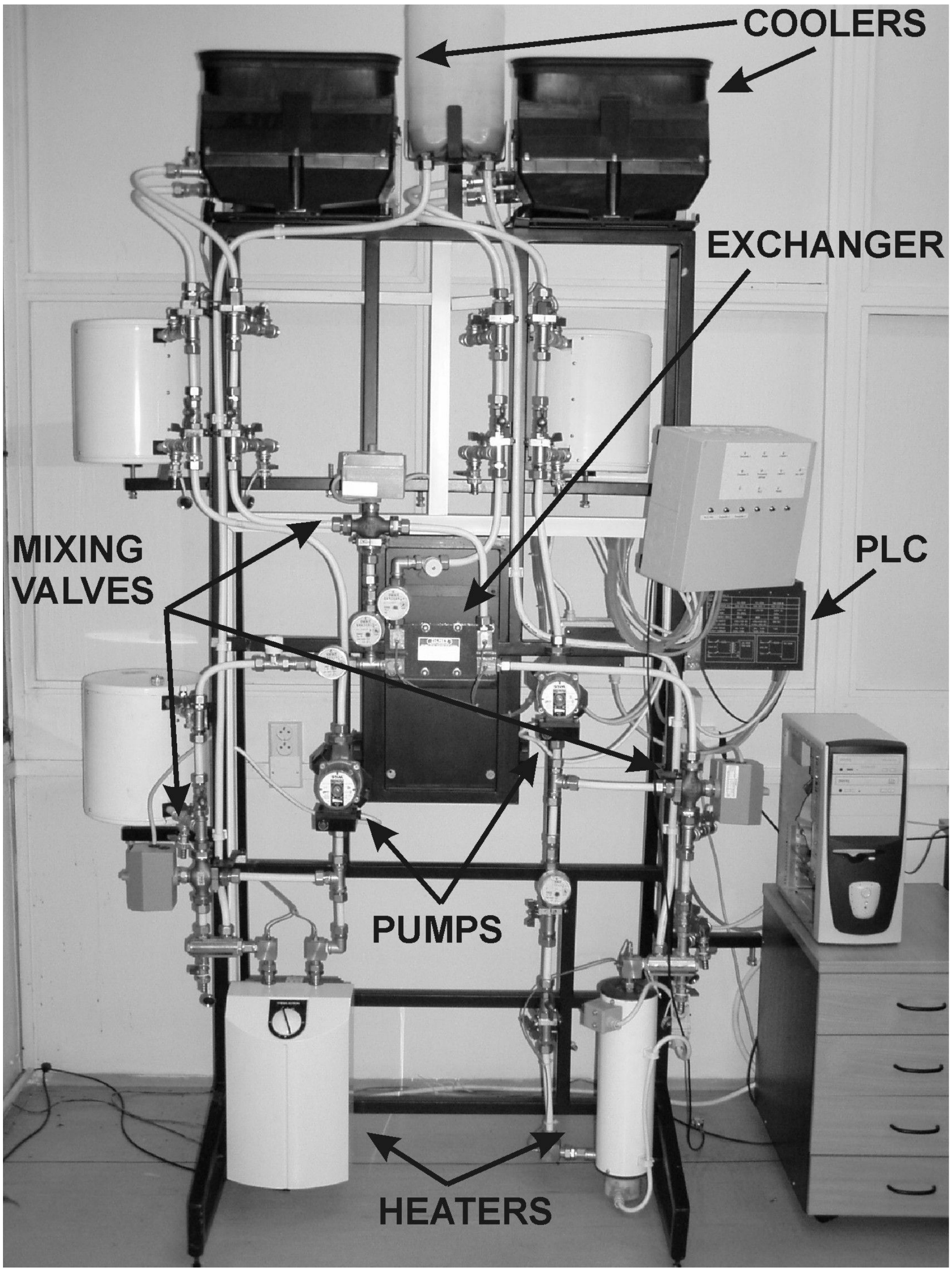

Fig. 1 Laboratory heating system built to test the TDS based modeling techniques and control design methods

2. LINEAR TIME DELAY SYSTEM

To make use of delay relations as one of the primary elements of model structure and to cover as wide as possible class of time delay systems the following convolution based system state description can be used

$$\frac{d\mathbf{x}(t)}{dt}=\int_0^T d\mathbf{A}(\tau)\mathbf{x}(t-\tau)+\int_0^T d\mathbf{B}(\tau)\mathbf{u}(t-\tau)$$
$$\mathbf{y}(t)=\mathbf{C}\mathbf{x}(t) \tag{1}$$

where τ is delay variable appearing in both the inputs **u** and state variables **x** and T is its maximum value, **y** are the system outputs and **C** is matrix of coefficients. The matrices $\mathbf{A}(\tau),\mathbf{B}(\tau)$ define the respective delay distributions of appropriate mutual interactions. Their elements are mostly not continuous but stepwise functions with the steps corresponding to the lumped delays. Stieltjes integrals in (1) help to cover both the lumped and distributed delays under a common general form of the model and result in a formally simple form of Laplace transform

$$s\mathbf{x}(s)=\mathbf{A}(s)\mathbf{x}(s)+\mathbf{B}(s)\mathbf{u}(s) \tag{2}$$

where

$$\mathbf{A}(s)=\int_0^T \exp(-s\tau)d\mathbf{A}(\tau),\ \mathbf{B}(s)=\int_0^T \exp(-s\tau)d\mathbf{B}(\tau) \tag{3}$$

if zero valued initial conditions are considered. Apparently $\mathbf{A}(s),\mathbf{B}(s)$ are s-multiples of the regular Laplace transforms of $\mathbf{A}(\tau),\mathbf{B}(\tau)$ and the product form of (2) saves most of the advantages of matrix calculus applications.

Unlike the state of a delay free linear system (matrices **A** and **B** are constant coefficient matrices), which is given by the vector of state variables $\mathbf{x}(t)$, the state of a TDS (1) is given by the function segment of the system state variables on the last system history interval

$$\boldsymbol{x}_t(\tau)=\mathbf{x}(t+\tau),\quad -T\le\tau\le 0 \tag{4}$$

and the state space is the Banach space of continuous real functions on the interval of length T, $\mathcal{C}=C([-T,\ 0],\mathrm{R}^n)$.

2.1 Example of TDS, model of laboratory heating system

As an illustration example of a TDS model, the model of a laboratory heating system is presented. The laboratory heating system has been built at the Department of Instrumentation and Control Engineering in order to test the control algorithms developed for TDS. The scheme of the laboratory system is sketched in Fig. 2. It consists of two heating circuits with the circulation of the heat medium (water) forced by two pumps (one in each circuit). The heat source of the system is an electric heater, located in the primary circuit. The heat exchange between the two circuits, which is controlled by the mixing valve, takes place in the multi-plate heat exchanger. The last subsystem component of the system is an air-water cooler located in the secondary circuit. The components of the system are connected by pipelines, which provide the most significant delays in the system. For detailed description of the system see (Vyhlídal, 2003).

The heat distribution on the plant is monitored by measuring the water temperatures by thermometers at four various places on the system, see Fig. 2, where ϑ_h is mixed up hot water temperature at the input of the exchanger, ϑ_a is exchanger outlet water temperature, ϑ_d, ϑ_c are air/water cooler inlet and outlet water temperatures, respectively. The control input of the system is the signal u controlling the mixing ration of the water flow rates coming from the heater and the exchanger in the valve, and thereby the temperature ϑ_h. As it is usual in control schemes of distributed heating systems, a slave control loop with P controller is used to control the actuator. The controlled variable of this loop is temperature ϑ_h and the reference value of this loop is $\vartheta_{h,set}$.

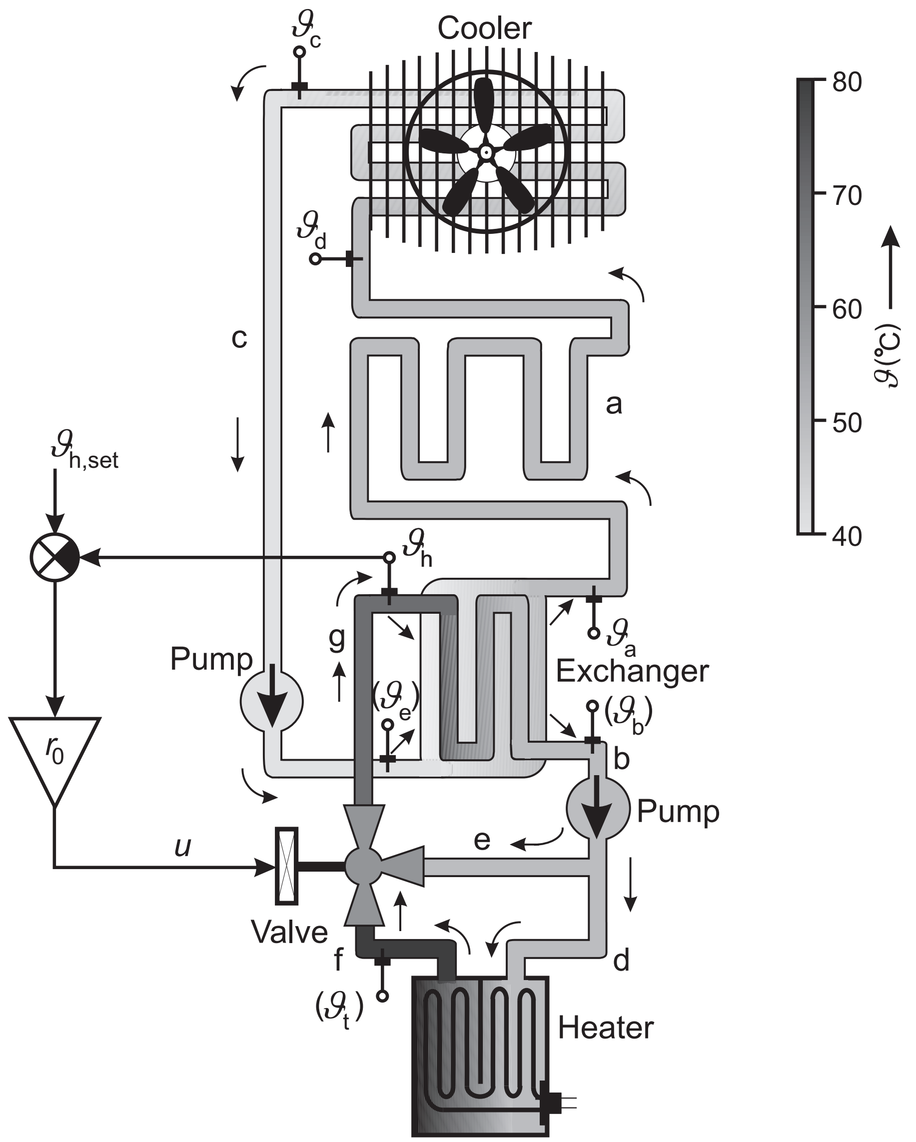

Fig. 2 Scheme of the laboratory heating system

Analysing the dynamics of the system in a vicinity of a selected operational point, a linear model has been built, see (Vyhlídal, 2003). The model consists of four functional differential equations

$$
\begin{aligned}
T_{\mathrm{h}}\frac{d\Delta\vartheta_{\mathrm{h}}(t)}{dt} &= -\Delta\vartheta_{\mathrm{h}}(t-\eta_h)+K_{\mathrm{b}}\Delta\vartheta_{\mathrm{a}}(t-\tau_{\mathrm{b}})+K_{\mathrm{u}}\Delta\vartheta_{\mathrm{h,\,set}}(t-\tau_{\mathrm{u}})\\
T_{\mathrm{a}}\frac{d\Delta\vartheta_{\mathrm{a}}(t)}{dt} &= K_{\mathrm{a}}\left[\Delta\vartheta_{\mathrm{h}}(t)-\frac{1+q}{2}\Delta\vartheta_{\mathrm{a}}(t)-\frac{1-q}{2}\Delta\vartheta_{\mathrm{c}}(t-\tau_{\mathrm{e}})\right]-\left[\Delta\vartheta_{\mathrm{a}}(t)-\Delta\vartheta_{\mathrm{c}}(t-\tau_{\mathrm{e}})\right]\\
T_{\mathrm{d}}\frac{d\Delta\vartheta_{\mathrm{d}}(t)}{dt} &= -\Delta\vartheta_{\mathrm{d}}(t)+K_{\mathrm{d}}\Delta\vartheta_{\mathrm{a}}(t-\tau_{\mathrm{d}})\\
T_{\mathrm{c}}\frac{d\Delta\vartheta_{\mathrm{c}}(t)}{dt} &= -\Delta\vartheta_{\mathrm{c}}(t-\eta_{\mathrm{c}})+K_{\mathrm{c}}\Delta\vartheta_{\mathrm{d}}(t-\tau_{\mathrm{c}})
\end{aligned}
\tag{5}
$$

Let us remark that in the model of the multi-plate heat exchanger (second equation in (5)) the logarithmic temperature gradient is approximated by the formula

$$
\Delta\overline{\vartheta} = \vartheta_{\mathrm{h}} - \frac{1+q}{2}\vartheta_{\mathrm{a}} - \frac{1-q}{2}\vartheta_{\mathrm{e}} \tag{6}
$$

where ϑ_{e} is the other inlet temperature of the exchanger and $q = m_2/m_1$, where m_1, m_2 are the flow rates in the parts of the exchangers, i.e., the flow rates in the heating circuits. The state variables of the model are chosen identical with the measured system outputs, i.e.

$$
\mathbf{x}(t) = \left[\Delta\vartheta_{\mathrm{h}}(t) \quad \Delta\vartheta_{\mathrm{a}}(t) \quad \Delta\vartheta_{\mathrm{d}}(t) \quad \Delta\vartheta_{\mathrm{c}}(t)\right]^T \tag{7}
$$

and the system input is the set-point value of the slave control loop $\Delta\vartheta_{h,set}(t)$. Applying the Laplace transform, the matrices of the model (5) are given as follows

$$
\mathbf{A}(s) = \begin{bmatrix}
\frac{-\mathrm{e}^{-\eta_{\mathrm{h}}s}}{T_{\mathrm{h}}} & \frac{K_{\mathrm{b}}\mathrm{e}^{-\tau_{\mathrm{b}}s}}{T_{\mathrm{h}}} & 0 & 0\\
\frac{K_{\mathrm{a}}}{T_{\mathrm{a}}} & \frac{-(1+0.5K_{\mathrm{a}}(1+q))}{T_{\mathrm{a}}} & 0 & \frac{(1-0.5K_{\mathrm{a}}(1-q))\mathrm{e}^{-\tau_{\mathrm{e}}s}}{T_{\mathrm{a}}}\\
0 & \frac{K_{\mathrm{d}}e^{-\tau_{\mathrm{d}}s}}{T_{\mathrm{d}}} & \frac{-1}{T_{\mathrm{d}}} & 0\\
0 & 0 & \frac{K_{\mathrm{c}}\mathrm{e}^{-\tau_{\mathrm{c}}s}}{T_{\mathrm{c}}} & \frac{-\mathrm{e}^{-\eta_{\mathrm{c}}s}}{T_{\mathrm{c}}}
\end{bmatrix}
$$

$$
\mathbf{B}(s) = \begin{bmatrix} \frac{K_{\mathrm{u}}\exp(-\tau_{\mathrm{u}})}{T_{\mathrm{h}}} & 0 & 0 & 0 \end{bmatrix}^{\mathrm{T}} \tag{8}
$$

The parameters of the model have been identified as follows: $T_{\mathrm{h}} = 14\mathrm{s}$, $K_{\mathrm{b}} = 0.24$, $K_{\mathrm{u}} = 0.39$, $\eta_{\mathrm{h}} = 6.5\mathrm{s}$, $\tau_{\mathrm{b}} = 40\mathrm{s}$, $\tau_{\mathrm{u}} = 13.2\mathrm{s}$, $T_{\mathrm{a}} = 3\mathrm{s}$, $K_{\mathrm{a}} = 1$, $\tau_{\mathrm{e}} = 13\mathrm{s}$, $q = 1$, $T_{\mathrm{d}} = 3\mathrm{s}$, $K_{\mathrm{d}} = 0.94$, $\tau_{\mathrm{d}} = 18\mathrm{s}$, $T_{\mathrm{c}} = 25\mathrm{s}$, $K_{\mathrm{c}} = 0.81$, $\eta_{\mathrm{c}} = 9.2\mathrm{s}$, $\tau_{\mathrm{c}} = 2.8\mathrm{s}$. As can be seen in Fig. 3, where the comparison of the measured and simulated step responses is shown, in the vicinity of the operational point, the model approximates the system dynamics very well.

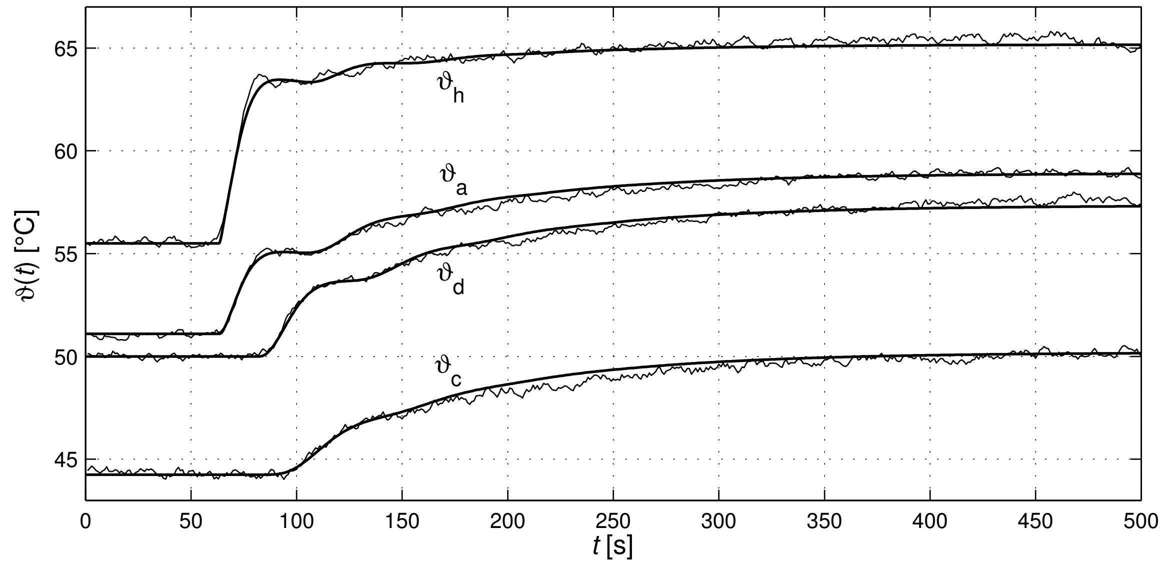

Fig. 3 Comparison of the step responses of the laboratory system (influenced by noise) and its TDS model given by (5) (smooth), $\Delta\vartheta_{\mathrm{h,set}}(50\mathrm{s}) = 20^{\circ}\mathrm{C}$

3. ALGORITHM FOR COMPUTING SPECTRUM OF TDS

The characteristic equation of the system (1)

$$M(s) = \det(s\mathbf{I} - \mathbf{A}(s)) = 0 \tag{9}$$

is transcendental with infinite spectrum of the roots determining the dynamics modes of TDS. An efficient assessment of a sufficiently large part of the infinite spectrum of a TDS is required in a large number of control design techniques for this class of systems. Since there are no general analytical formulas available for computing the zeros of any quasi-polynomial of form (9), a numerical method has to be used.

In (Vyhlídal and Zítek, 2003) an algorithm is introduced to compute the spectrum of a TDS located in a closed region of the complex plane. The algorithm, which is based on mapping the quasi-polynomial, proved efficient in locating large number of roots. The basic idea of the quasi-polynomial mapping based rootfinder (QPMR) is given as follows.

Consider functions $R(\beta,\omega) = \mathrm{Re}(M(\beta+\mathrm{j}\omega))$ and $I(\beta,\omega) = \mathrm{Im}(M(\beta+\mathrm{j}\omega))$. Then the characteristic equation (9) can be split into

$$R(\beta,\omega)=0 \tag{10}$$

$$I(\beta,\omega)=0 \tag{11}$$

These equations determine the intersection contours of the surfaces described by $R(\beta,\omega)$ and $I(\beta,\omega)$, respectively, with the s-plane. Mapping these zero-level contours, the roots of (9) are given as the intersection points of contours described implicitly by (10) and (11). Obviously, the contours $R(\beta,\omega) = 0$ and $I(\beta,\omega) = 0$ can be expressed analytically only for the most simple quasi-polynomials. In general, a numerical contour plotting algorithm is to be used to map the contours, e.g., the level curve tracing algorithm. In (Vyhlídal and Zítek, 2006) an advanced version of QPMR algorithm is described. In this modification of the method, Bellman and Cooke (1963) analytical approach for determining the strips in the complex plane where the root chains are located is utilized to reduce significantly the area of the selected region to be scanned for the roots.

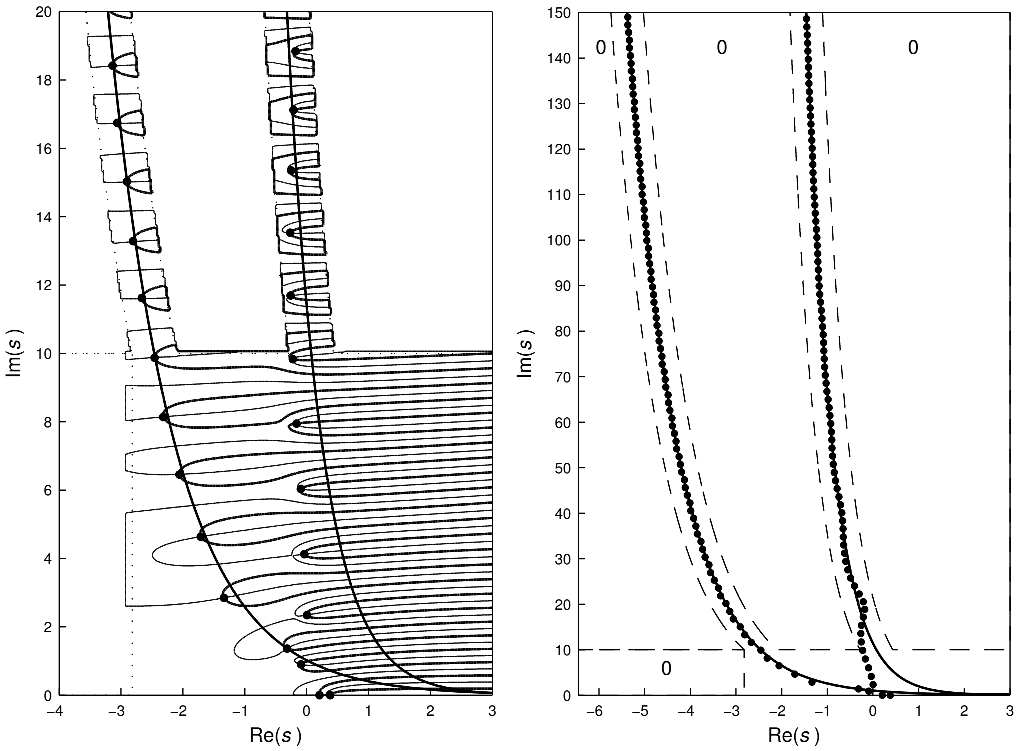

Fig. 4 Left - map of the curves $R(\beta,\omega)$ =0 (thick) and $I(\beta,\omega)$ =0 (thin) of the quasi-polynomial (12) of a retarded TDS, black dots - roots, bold solid lines – asymptotic exponentials of the root chains.

Right - spectrum of the quasi-polynomial (12), solid line – asymptotic exponentials of the root chains, dashed lines - boundaries of the regions which have not been mapped, black dots- roots

In Fig. 4 an illustration example of the spectrum of the quasi-polynomial

$$
\begin{aligned}
M(s) = {} & 0.01s^6 + 0.03s^5 - 0.2s^4 + 0.7s^3 + 1.2s^2 - 0.2s + 0.3 + \\
& + (1.6s^4 + 1.2s^3 + 2.3s^2 + 0.3s - 1.5)\mathrm{e}^{-0.31s} + \\
& + (-1.3s^4 + 1.1s^3 + 0.5s^2 - 0.1s - 0.4)\mathrm{e}^{-3.53s} + \\
& + (-0.6s^2 + 1.3s - 0.3)\mathrm{e}^{-4.09s} + (-0.4s + 0.7)\mathrm{e}^{-5.88s} + 1.4\mathrm{e}^{-7.24s}
\end{aligned}
\tag{12}
$$

computed using such a modification of QPMR is shown. To sum up, the mapping based rootfinding algorithm allows us to assess several hundreds of roots of TDS in a very reasonable manner. For example, In (Olgac, et al., 2006) the QPMR is used for assessing the stability regions in the domain of delays. QPMR is also a crucial tool in the application of pole placement method in TDS, which is described in the next section.

4. DIRECT POLE PLACEMENT IN TDS

In case of a linear delay free system, all the system modes can be determined via the pole placement method by the proportional feedback from the state variables

$$u(t) = -\,\mathbf{K}\,\mathbf{x}(t) \tag{13}$$

called the state feedback. However, if applied to system (1), the feedback (13) does not accomplish a state feedback. It is due to fact that the state of (1) is given by (4) and not by the vector $\mathbf{x}(t)$ as in case of delay free systems. In spite of this, feedback (13) has proved to be an efficient tool to design robustly stable dynamics for TDS, see e.g. (Zítek and Vyhlídal, 2000), (Vyhlídal, 2003). After closing feedback (13), the characteristic equation of the feedback system is as follows

$$M(s) = \det\left[\, s\,\mathbf{I} - \mathbf{A}(s) + \mathbf{B}(s)\mathbf{K} \,\right] = 0 \tag{14}$$

Suppose the original spectrum of system matrix $\mathbf{A}(s)$ is $\mathrm{Sp}(\mathbf{A}(s)) = \{\lambda_i\}$, $i = 1..\infty$ and the unaffected characteristic quasi-polynomial is $M_0(s)$. After adding feedback (13) the original quasi-polynomial acquires the form $M(s)$ in (14) with a new spectrum $\mathrm{Sp}(\mathbf{A}(s) - \mathbf{B}(s)\mathbf{K}) = \{\sigma_i\}, i = 1..\infty$. Since the quasi-polynomial (14) is linear with respect to $\mathbf{K}$, the following relationship holds between the original $M_0(s)$ and the feedback system quasi-polynomial $M(s,\mathbf{K})$

$$M(s,\mathbf{K}) = M_0(s) + \sum_{j=1}^{n} \frac{\partial M(s,\mathbf{K})}{\partial K_j} K_j \tag{15}$$

where the gradient derivatives with respect to K_j, i.e., sensitivity functions, are independent of **K**. Obviously these derivatives are variable in s too, owing to the exponential functions in $M(s)$.

Consider a group of dominant $M_0(s)$ zeros $s = \lambda_i$, $i = 1,2,...$ from the original system spectrum which provide the system with undesired properties. The aim is to shift the dominant zeros into prescribed new positions $s = \sigma_i, i = 1,2,...,$ which are to become the zeros of the designed $M(s)$, i.e. $M(\sigma_i) = 0$ and which endow the system with more favourable dynamics. For any prescribed poles σ_i the following relationship holds

$$M(\sigma_i, \mathbf{K}) = 0 = M_0(\sigma_i) + \sum_{j=1}^{n} K_j \left[\frac{\partial M(s, \mathbf{K})}{\partial K_j} \right]_{s=\sigma_i} \tag{16}$$

i.e., a set of linear algebraic equations with the unknown parameters $K_1, K_2, ..., K_n$. The desired new positions of σ_i can be prescribed for no more than n zeros at the most, where n is the number of feedback coefficients K_i. Apparently the desired eigenvalue positions are to be prescribed with respect to λ_i, $i = 1,2,...$ constituting the group of the significant (dominant) system poles. It is of little sense to assign the insignificant system eigenvalues because they cannot affect the actual system behaviour. In (Zítek and Vyhlídal, 2002) a method for pole shifting via (13) has been designed. First, rightmost spectrum of the poles of system (1) is computed using QPMR. Then the significance of the poles of the original system is evaluated based on residues of partial fractions of the Heaviside expansion applied to system (1), see also (Zítek and Vyhlídal 2004). In the second step, the selected dominant poles are shifted to desired positions and the feedback gain **K** is computed from the set of equations (16). Finally the spectrum of the feedback system is computed using QPMR and the significance of the poles is checked again. If the shifted poles are the most significant poles, i.e., the poles determining the system dynamics, the synthesis has been successful and the computed feedback gain can be used. On the other hand, if the shifted poles are not the most significant poles, the whole procedure is to be repeated again.

4.1 Application of direct pole placement to the laboratory heating system

In the framework of the research presented, not only the theoretical work has been done, but the state variable feedback control has also been tested practically on the laboratory heating system described in Section 2.1. Consider the model given by four functional differential equations (5) plus one ordinary differential equation

$$\frac{dI(t)}{dt} = \Delta\vartheta_{\mathrm{c,set}}(t) - \Delta\vartheta_{\mathrm{c}}(t) \tag{17}$$

which is added in order to accomplish the astatism of the open loop system.

In Fig. 6, left, the rightmost spectrum of poles of such a system is shown. The system is to be controlled using the feedback (13), which is of the form

$$\Delta\vartheta_{h,set}(t) = -\begin{bmatrix} K_1 & K_2 & K_3 & K_4 & K_5 \end{bmatrix}\begin{bmatrix} \Delta\vartheta_{\mathrm{h}}(s) & \Delta\vartheta_{\mathrm{a}}(s) & \Delta\vartheta_{\mathrm{d}}(s) & \Delta\vartheta_{\mathrm{c}}(s) & I(s) \end{bmatrix}^T \tag{18}$$

Prescribing one real pole $\sigma_1 = -0.07$ and double complex conjugate poles $\sigma_{2,3} = \sigma_{4,5} = -0.03 \pm 0.06\mathrm{j}$, from the set of equation of the form (16), the following feedback gain results $\mathbf{K} = \begin{bmatrix} -0.298 & 4.091 & 7.690 & 4.576 & -0.352 \end{bmatrix}$. From the spectrum of the feedback system shown in Fig. 6 right, we can see that the prescribed poles are the poles with the smallest modules and they are likely

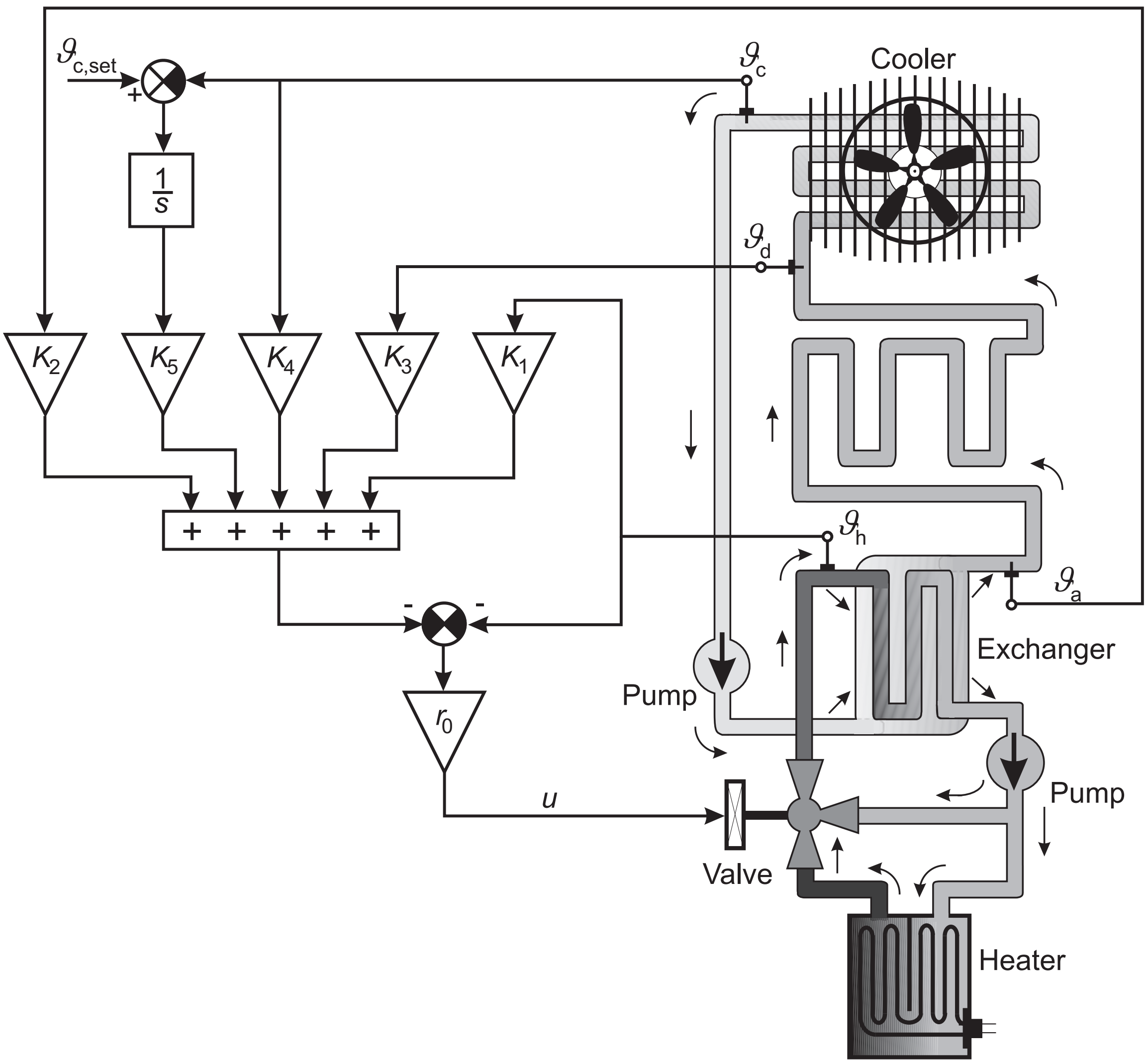

Fig. 5 Scheme of the state variable feedback control of the laboratory heating system

to be the dominant poles. In Fig. 7, the set point response of the feedback system is shown measured on the heating system and compared with the simulated set-point response. As can be seen from the comparison, the feedback system dynamics has been assigned well not only to the model but also to the controlled plant.

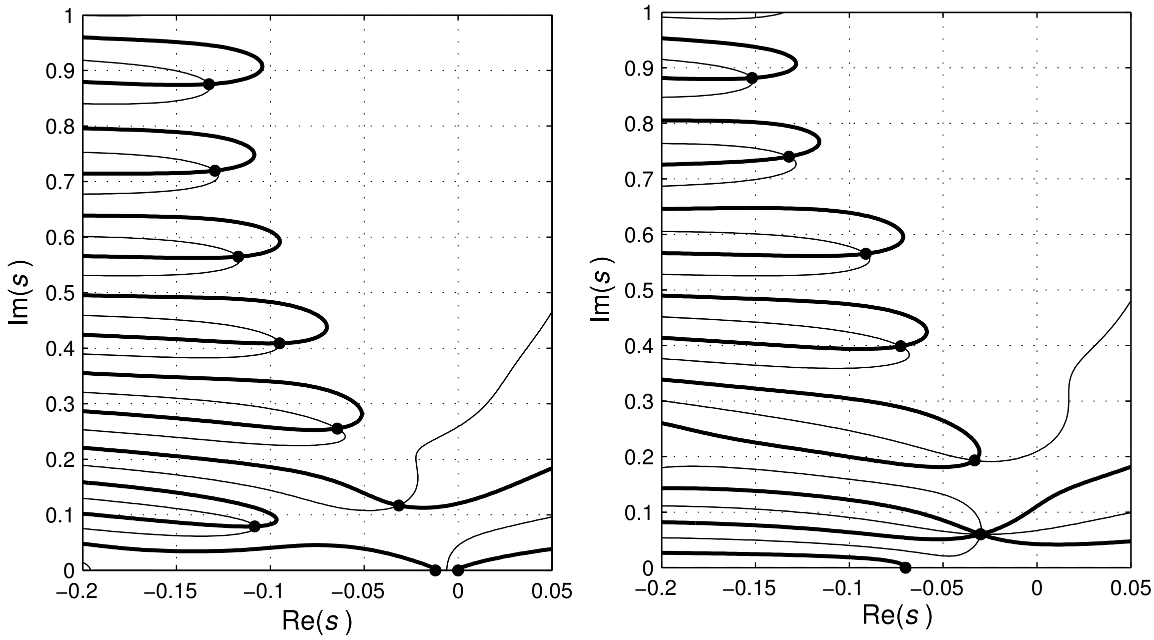

Fig. 6 Spectrum computed using QPMR, left – the spectrum of the heating system plus integrator, right – spectrum of the heating system with the state variable feedback

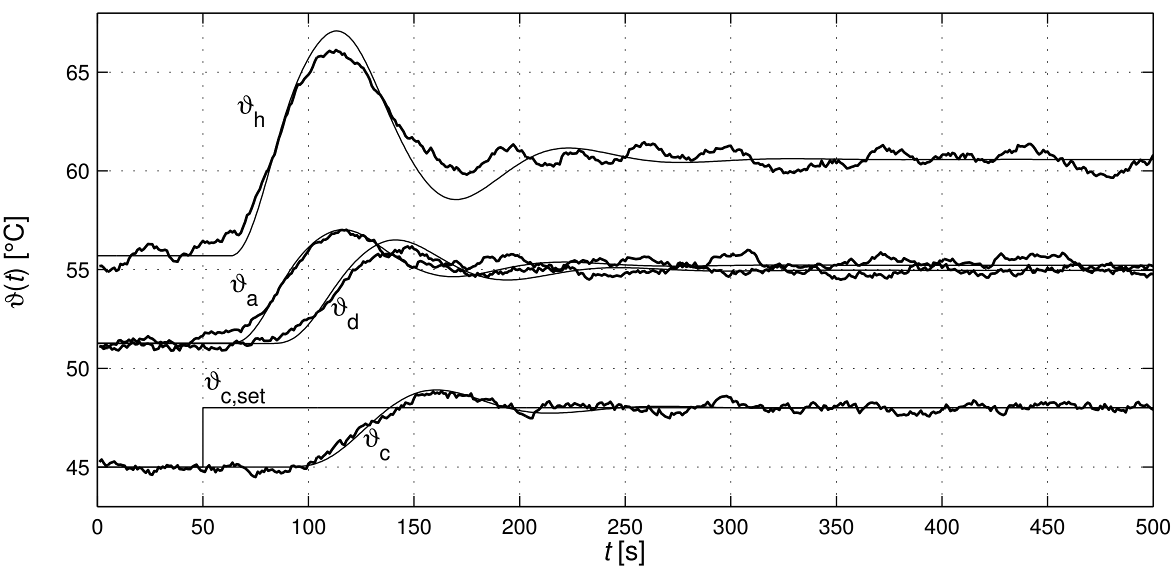

Fig. 7 Comparison of the simulated (smooth) and the measured (influenced by noise) set-point responses of the laboratory heating system

5. CONTINUOUS POLE PLACEMENT APPLIED TO NEUTRAL SYSTEM

An automated iteration pole placement procedure for TDS of the form (1) (retarded system) has been designed by Michiels, et. al. (2002) and is known as continuous pole placement. In this procedure, the poles are continuously shifted to the left via small replacements at each iteration. In (Vyhlídal and Michiels, 2004) and (Michiels and Vyhlídal, 2005) the continuous pole placement method is extended towards the TDS of the neutral form

$$\frac{d\mathbf{x}(t)}{dt} = \sum_{i=1}^{N}\left[\mathbf{H}_i \frac{d\mathbf{x}(t-\eta_i)}{dt}\right] + \int_0^T d\mathbf{A}(\tau)\mathbf{x}(t-\tau) + \int_0^T d\mathbf{B}(\tau)\mathbf{u}(t-\tau) \qquad (19)$$

The complicacy in application of pole placement method in (19) arises from the fact that features of the spectrum of the neutral system (19) are completely different than the features of the spectrum of the retarded system (1). At least a part of the spectrum of the neutral system is asymptotic to the spectrum of the associated difference equation

$$\mathbf{x}(t) = \sum_{i=1}^{N}\mathbf{H}_i\mathbf{x}(t-\eta_i) \qquad (20)$$

which tends to be distributed in a finite width vertical strip of the complex plane. Moreover, the spectrum can be sensitive to even infinitesimal changes in the delays, (Hale and Verduyn Lunel, 1993). Therefore, as a preliminary step of the continuous pole placement algorithm, a procedure to determine the smallest safe upper bound on the real parts of the spectrum of the associated difference equation C_D is designed in (Vyhlídal and Michiels, 2004) and (Michiels and Vyhlídal, 2005). As it has been proved in the mentioned articles, C_D is a single zero of the equation

$$f(c,\eta)-1, \text{ where } f(c,\eta) = \max\left\{ r_\sigma\left(\sum_{k=1}^{N}\mathbf{H}_k \mathrm{e}^{-c\eta_k}\mathrm{e}^{\mathrm{j}\theta_k}\right) : \theta_k \in [0,2\pi]\right\} \qquad (21)$$

where r_σ denotes the spectral radius. It results from this procedure that the systems which have $C_D \geq 0$ cannot be safely stabilized by feedback (13). On the other hand if $C_D < 0$, the continuous pole placement can be used to stabilize or improve the dynamics of the neutral system.

5.1 Application example

Consider neutral system (19) given by the matrices

$$\mathbf{H}_1 = \begin{bmatrix} 0 & 0.2 & -0.4 \\ -0.5 & 0.3 & 0 \\ 0.2 & -0.7 & 0.1 \end{bmatrix}, \mathbf{H}_2 = \begin{bmatrix} -0.3 & -0.1 & 0.2 \\ 0 & 0.2 & -0.1 \\ 0.1 & 0 & 0.4 \end{bmatrix}, \mathbf{A} = \begin{bmatrix} -0.6 & 0.8 & -1 \\ -2.1 & 2.1 & -2.6 \\ 0.3 & 1.7 & -3.7 \end{bmatrix}, \mathbf{B} = \begin{bmatrix} 0.3 \\ -0.2 \\ 0.1 \end{bmatrix}$$

and by delays $\eta_1 = 0.61, \eta_2 = 1.8$ and the single input delay $\tau = 2.7$. In Fig. 8, left, the spectrum of the system, computed using QPMR is shown. As can be seen, the system is unstable with a couple of complex conjugate unstable poles. First, evaluating numerically $f(c, \eta)$, the safe upper bound is computed as $C_D = -0.1446 < 0$, which implies that the neutral system can be stabilized. Applying the continuous pole placement procedure, see its evolution in Fig. 9, the feedback gain is computed as $\mathbf{K} = [-0.6256 \;\; 0.3046 \;\; -0.5169]$, which safely stabilizes the neutral system, see the feedback system spectrum in Fig. 8, right.

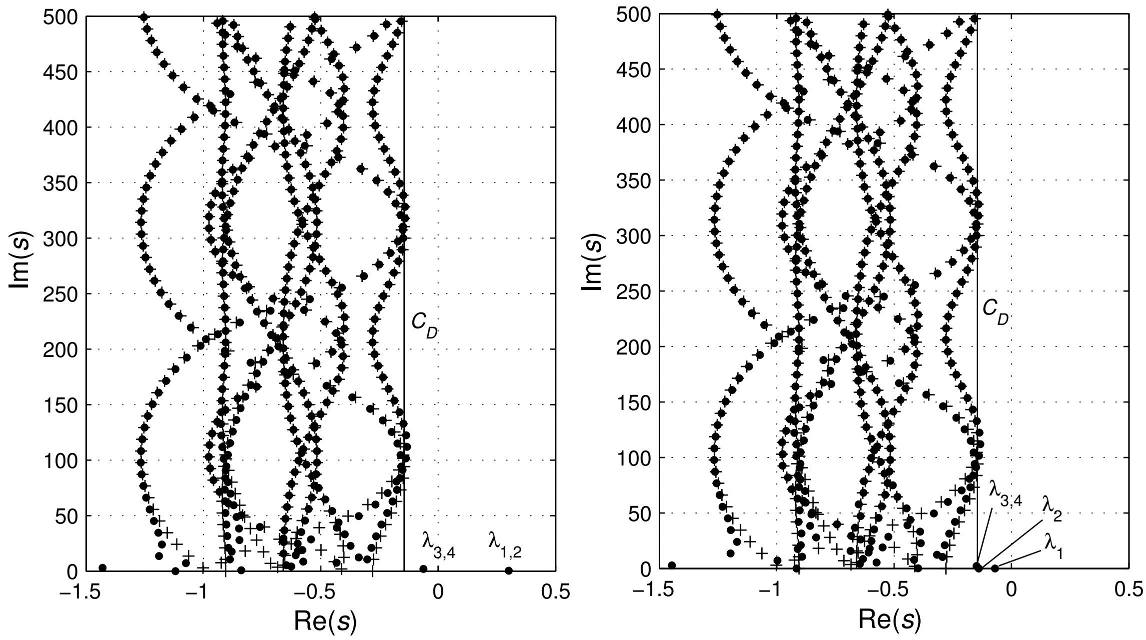

Fig. 8 Spectra of the neutral system (black dots) and the associated difference equation (crosses), left – without state variable feedback, right – with state variable feedback

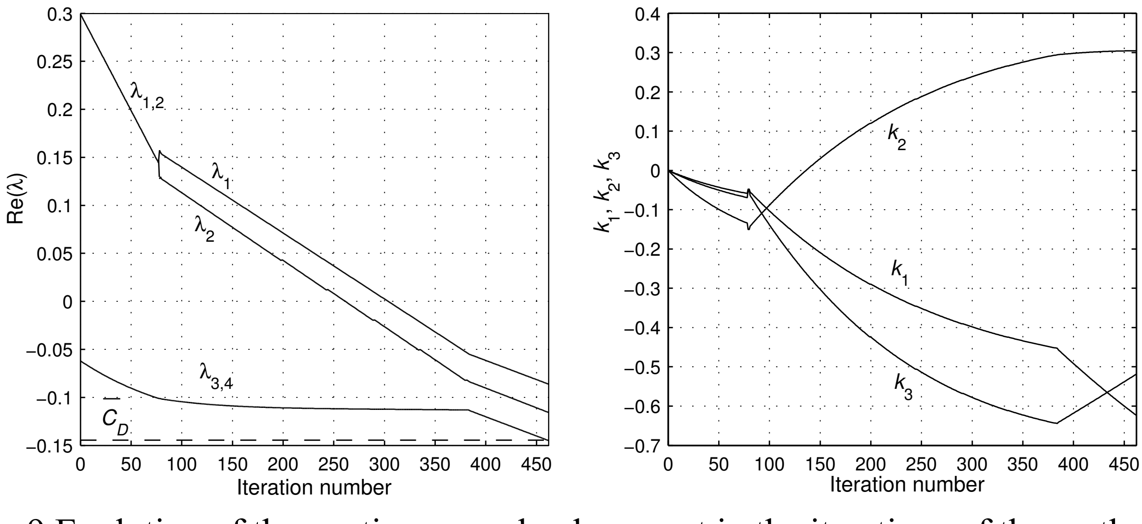

Fig. 9 Evolution of the continuous pole placement in the iterations of the method

6. FIRST ORDER TDS MODEL IN CONTROL DESIGN

As has been shown in (Vyhlídal and Zítek, 2001), (Zítek and Vyhlídal, 2003) the model with one integrator and two delays

$$T\frac{\mathrm{d}y(t)}{\mathrm{d}t}+y(t-\phi)=Ku(t-\tau) \tag{22}$$

with the transfer function

$$G(s)=\frac{K\exp(-s\tau)}{Ts+\exp(-s\phi)} \tag{23}$$

where K is the static gain, T is the time constant and ϕ,τ are time delays, is suitable and sufficient for describing large frequency range of the dynamics of a great deal of real plants which are usually described by higher order models. In the mentioned articles, a method for identifying parameters of (22) is designed based on step response and relay feedback test (Åström and Hägglund, 1998). Consider, the static gain K and the input delay τ are assessed from the step response, time constant T and the other delay ϑ can be computed from

$$\phi=\frac{\pi-\arccos\{Kk_u\cos(\omega_u\tau)\}}{\omega_u} \tag{24}$$

$$T=\frac{\sin(\omega_u\phi)-\tan(\omega_u\tau)\cos(\omega_u\phi)}{\omega_u} \tag{25}$$

where ω_{u} and $k_u=4u_a(\pi y_a)^{-1}$ (u_a, y_a are the amplitudes of system input and output oscillations, respectively) result from the relay feedback test.

Applying the Internal Model Control (IMC) scheme (Morari, Zafiriou, 1989) to model (23) (Vyhlídal and Zítek, 2001) the resulting Anisochronic IMC controller (AIMC) acquires the following form

$$R(s)=\frac{Ts+\exp(-s\phi)}{K(T_f s+1-\exp(-s\tau))} \tag{26}$$

where the parameters K, T, τ and ϕ are taken from the model (23) and the time constant of the filter T_f is the optional parameter, by which the closed loop dynamics is adjusted, because

$$G_{wy}(s)=\frac{R(s)G(s)}{1-R(s)G(s)}=\frac{\exp(-s\tau)}{T_f s+1} \tag{27}$$

In the ideal case where model (22) is identical with the real plant dynamics, the controller compensates the system dynamics completely and the dynamics of the feedback system are given by the single mode $s=-1/T_f$.

6.1 Implementation of the anisochronic controller on PLC

The AIMC algorithm given by (26) has been discretized and implemented on a programmable (logic) controller PLC Tecomat NS950 (Teco, Kolín) with analog input and output modules. The algorithm of the controller, see its schema in Fig. 10, has been designed taking into account the usual implementation issues: smooth automatic/manual switching, smooth AIMC/PID switching, saturation of the control variable, saturation of the increment of the control variable (also in the manual mode), antiwind-up, three modes of the set-point variable changes (step, ramp and first order filter), automatic assessing of the sampling period with automatic resizing the memory registers for delaying the variables, relay based identification algorithm with computing all the model parameters.

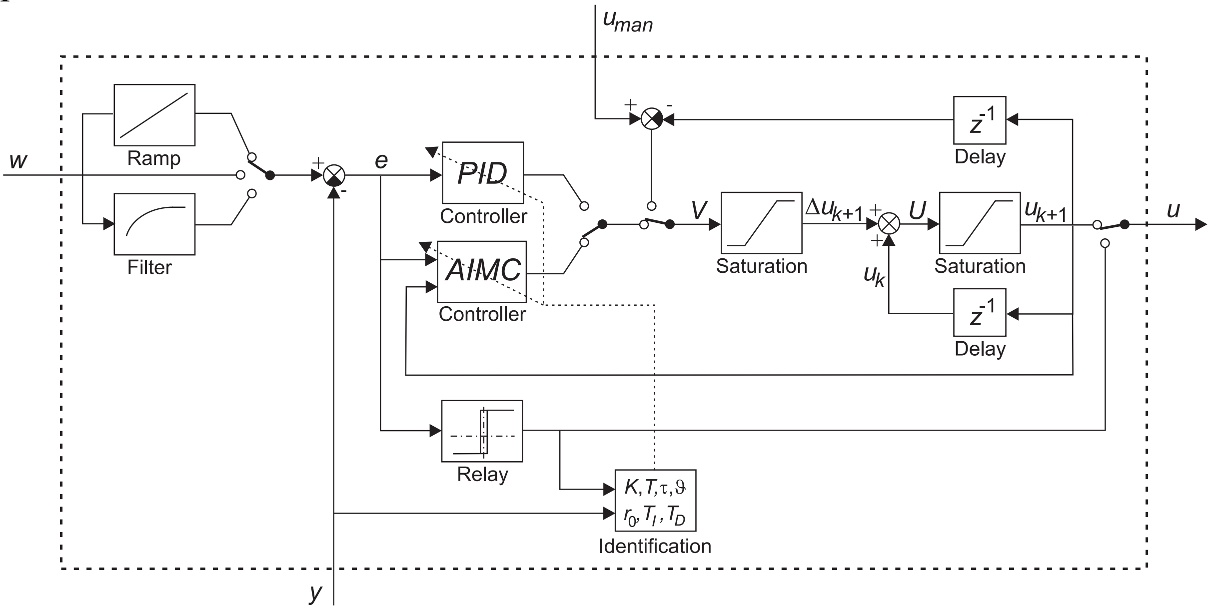

Fig. 10 Controller algorithm scheme of the implementation on PLC

The AIMC algorithm implementation on PLC Tecomat has been tested on a part of the laboratory heating system shown in Fig. 1. The system here is a heating circuit with the forced circulation of water by a pump, see Fig. 11. The main parts of the system are a flow-type heater and a cooler which are connected by pipelines. The system output variable (controlled variable) is the outlet temperature of the heater and the system input variable (control variable) is the heat performance of the heater. The temperature is measured by a thermometer. The control variable u generated by PLC analog output module controls the performance of the heater. Obviously, the outlet temperature depends on the heater inlet temperature, heater performance and water flow through the heater. If the water flow and the inlet temperature are considered constant, the changes of the outlet temperature of the heater from the operational point temperature are caused by the changes of the heater performance.

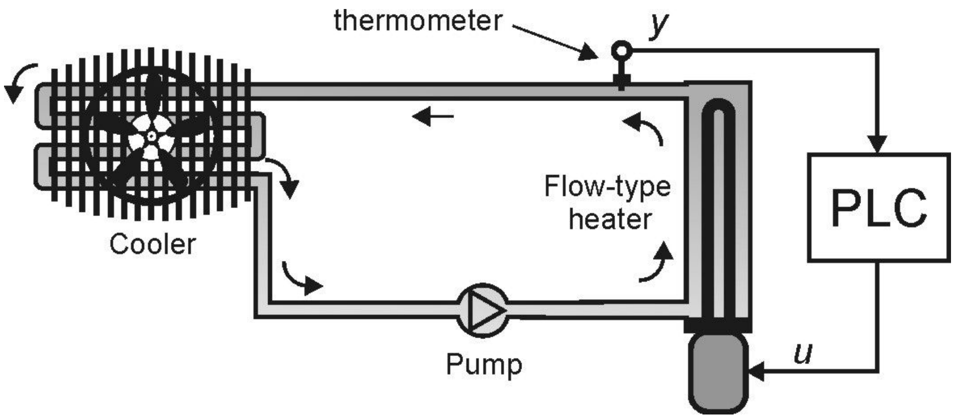

Fig. 11 Scheme of the control set up

First step in assessing the parameters of the anisochronic controller (26) consists in identification of the parameters of the model (22) chosen to describe the dynamics relationship between the heating performance and the outlet temperature in a vicinity of a chosen operational point $y \approx 75$ºC ($u \approx 62\%$). The static gain K=0.25 and the input delay $\tau = 13$s are estimated from the step response of the system, see Fig. 12, left, while the time constant T=29.6s and the delay ϕ=11.2s are computed from the critical parameters of the closed loop $k_u = 4u_a/\pi y_a = 10.9$ and $\omega_u = 1/T_u = 0.125\text{s}^{-1}$, which are estimated from the relay feedback experiment, see Fig. 12, right, (u_a, y_a are amplitudes of the control and controlled variable, respectively, and T_u is the period of the relay oscillations). The PLC program has subroutines for automatic estimation of the critical parameters k_u and ω_u and computing the parameters T and ϕ, as well as estimating the input delay τ. According to the identified parameters, the optional parameter is chosen T_f=40s, which is large enough to guarantee non-oscillatory closed loop dynamics and which is not too large to cause slow set-point tracking and disturbance rejection.

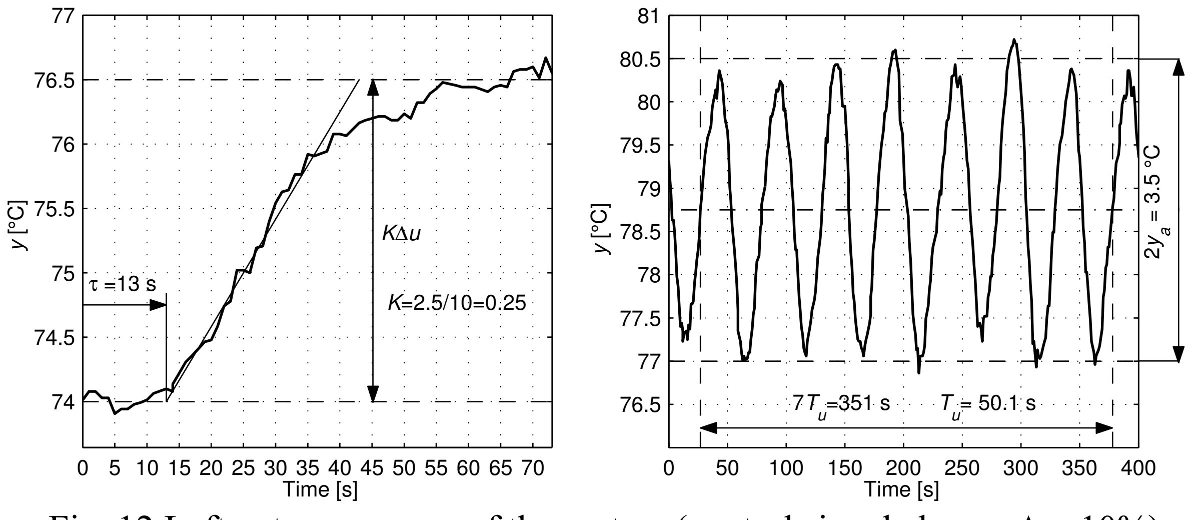

Fig. 12 Left - step response of the system (control signal change Δu=10%), right - oscillations of the relay feedback test (amplitude of the control signal u_a=15%).

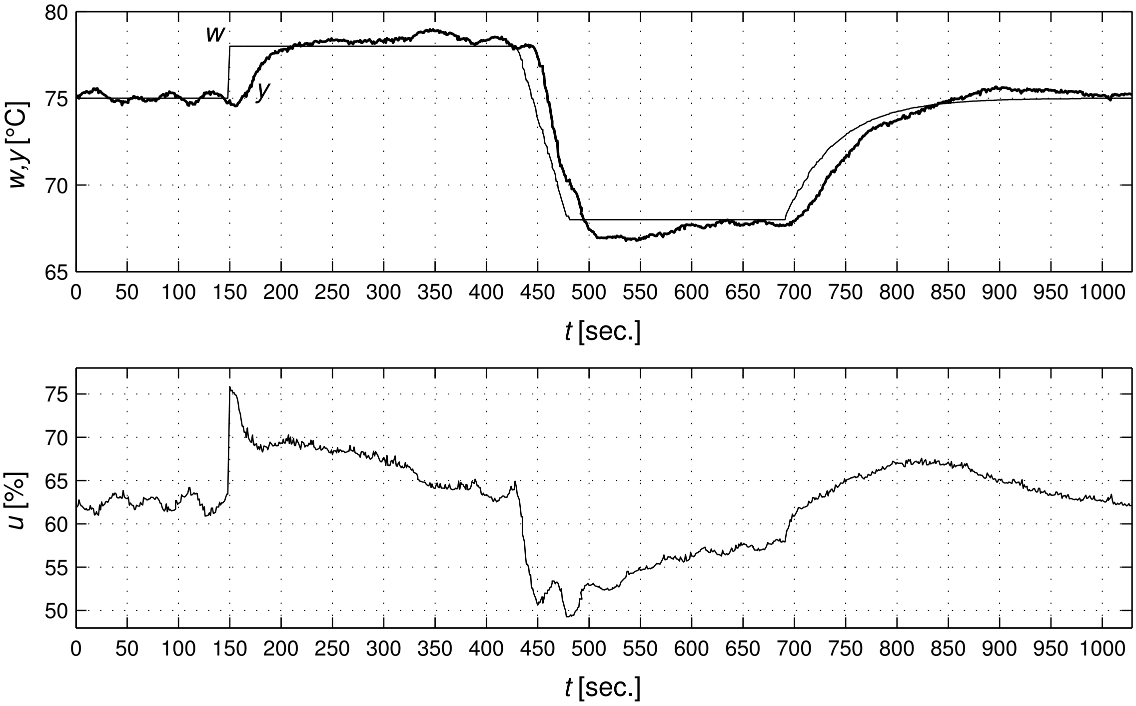

Fig. 13 Set-point tracking by AIMC controller with the parameters K=0.25, τ= 13s, T=29.6s, ϕ=11.2s and T_f=40s. w – set-point (desired value of the controlled variable), y – controlled variable

The results of the control by the PLC with the implementation of AIMC are shown in Fig 13 where the set-point variable tracking accomplished by AIMC with the parameters assessed above can be seen. Almost ideally shaped step set-point response (set-point variable is changed from w=75ºC to w=78ºC, showing that the closed loop dynamics is really close to the first order dynamics with time constant T_f=40s as it is predicted in (27)) proves the very good features of the anisochronic control algorithm. Also the ramp and first order filter set-point responses, shown also in Fig. 13, reveal good features of the closed loop dynamics and proves quality of the implementation.

7. CONCLUSIONS

The main advantage of time delay based modelling approach presented in this text is that it allows us to build such models of systems with distributed parameters which have the state variables identical with the measured system outputs. For example, the mathematical model of the laboratory heating system presented in Section 2.1, consists of a set of only four first order functional differential equations with the state variables which represent the measured temperatures. Let us point out that the time delays have been used not only to describe the transportation phenomenon in the pipelines, but also in the models of subsystems with capacities and distributed parameters (cooler, heater and heat exchanger). Since all the state variables of the heating system are measured, no observer is needed and implementation of the state variable feedback controller is easy and straightforward, as it has been shown in Section 4. However, since the time delay mathematical model is functional with infinite spectrum of eigenvalues, special mathematical tools have to be used in the control design. One of them is the algorithm for computing large parts of the spectrum of TDS presented in Section 3.

Apart from the state space models, time delay models can also be used to describe input-output dynamics. As an example of such a model, the first order model with two time delays is presented in Section 6. In spite of its first order, the model can be used to describe systems conventionally described by a higher order model. The control algorithm has been implemented on a programmable controller PLC Tecomat NS950 (Teco, Kolín) and successfully tested on the laboratory heating system.

To sum up, results of both theoretical and applied research are presented. Due to complexity of the theory of time delay systems, there are many open problems which provide broad potential for further research. Moreover, so far, the majority of the results achieved in the research focused on time delay systems have remained on the theoretical platform, which opens large space also for practically oriented research.

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LIST OF SYMBOLS

Abbreviations

IMC	internal model control
AIMC	anisochronic internal model control
TDS	time delay system(s)
QPMR	quasi-polynomial mapping based rootfinder
PLC	programmable logic controller

Symbols

$\mathbf{A}(\tau)$	functional matrix of system dynamics
$\mathbf{A}(s)$	Laplace transform of $\mathbf{A}(\tau)$
$\mathbf{B}(\tau)$	system input functional matrix
$\mathbf{B}(s)$	Laplace transform of $\mathbf{B}(\tau)$
$\mathcal{C}$	Banach space
$\mathbf{C}$	matrix of system outputs
C_D	safe upper bound of the spectrum
$G(s)$	transfer function of the model approximated real plant dynamics
$G_{wy}(s)$	transfer function of the closed loop system
$\mathbf{H}_i$	matrices of the difference equation associated with the neutral system
$\mathbf{I}$	identity matrix
$\mathrm{Im}(\cdot)$	imag part of $\cdot$
$\mathbf{K}$	feedback matrix
$M(s)$	characteristic function of the system
R	real space
$R(s)$	transfer function of the controller
$\mathrm{Re}(\cdot)$	real part of $\cdot$
s	complex variable, operator of Laplace transform
t	time
$\mathbf{u}(t)$	vector of system input variables
$\mathbf{u}(s)$	Laplace transform of $\mathbf{u}(t)$
$\mathbf{x}(t)$	vector of system state variables
$\mathbf{x}(s)$	Laplace transform of $\mathbf{x}(t)$
$\boldsymbol{x}_t$	state of TDS at time t
$\mathbf{y}(t)$	vector of system output variables
ϑ	temperature
η, ϕ	time delays
λ_i	poles of the system
σ_i	prescribed spectrum of poles
τ	delay variable, time delay

CURRICULUM VITAE

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Tomáš Vyhlídal was born in Dačice, on October 22, 1974. He received M.Sc. in Automatic Control and Engineering Informatics in 1998 and Ph.D. in Control and Systems Engineering in 2003, both from the Faculty of Mechanical Engineering, Czech Technical University in Prague and both under supervision of Prof. Ing. Pavel Zítek, Dr.Sc.. Since 2000, he has been with the Centre for Applied Cybernetics, CTU in Prague. His research interest include mathematical modeling and control system theory, focused mainly on the analysis and control of time-delay systems, modeling and control of heating systems and microclimate control.

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From 2002 to 2004 he was a member of the research team of the grant project (GAČR) "Microclimate control in historical interiors". Recently he has become a member of the research team of a large EU project "Smart technologies for stress free air travel".

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In 2001, in the framework of his doctoral program, he spent six months at the Department of Aerospace Engineering in University of Glasgow, Scotland. In 2005, he spent one month at the Department of Mechanical Engineering in University of Connecticut, USA. Besides the research work, he gave there an invited lecture "Spectrum based analysis and control design of a time delay system" in Fall 2005 seminar series.

He is married.