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Navrhování parametrů stavební konstrukce pomocí teorie tolerancí

Design of building structure parameters by means of theory of tolerances

Summary

The presented publication is about the application of the theory of tolerances in the design of the building structure by means of the thermal dynamic network models.

In the first part it is explained the importance of computer simulations for the design of the systems and especially for the design of the building structures. The standard approach and consequently new approach based on presented methods in next paragraphs is described.

Next part describes the theory of the thermal dynamic models built as the network.. The models include thermal resistances, thermal capacitances and sources of heat flows. Models employ both lumped and distributed parameters. Distributed parameters elements ensure exact outputs in the case of heavyweight structures. The models allow us to use window elements and automatically and manually controlled heat sources. The solution is based on the use of Laplace and fast Fourier transform. The simulation environment has been developed during last years. The program makes possible to create any model and works with the periodic changes of the load variables. The outputs from the simulation have been compared to the measured values.

The main part of theory of tolerance is the sensitivity analysis. In this case, it is the sensitivity analysis of the transfer function. The equations are presented for the calculation of the relative differential sensitivities of the model elements. The process of the calculation is demonstrated in the case of sensitivity analysis of the model that describes the changes of internal air temperature according to changes of the external air temperature. The most sensitive is the window element and the thermal transfer conductance between interior and surrounding building structures. Next example shows the application of theory of tolerances with the goal to find immediately the model parameters and building structure parameters for the desired changes of the internal air temperature in summer period.

The described method demonstrates the potential of the methods based on the tolerance theory. The restrictions in using these methods are also described in the end.

Souhrn

Práce je zaměřena na aplikaci teorie tolerancí při navrhování stavební konstrukce pomocí tepelných dynamických síťových modelů.

V první části je vysvětlen význam počítačových simulací pro navrhování systémů a zejména stavebních konstrukcí. Je zde uveden standardní postup využívající iterační proces a dále je popsán nový postup vycházející z níže popsaných metod.

Další část se věnuje teorii tepelných síťových modelů sestavených z tepelných odporů, tepelných kapacit a zdrojů tepelných toků. Modely využívají jak soustředěné parametry, kdy pracujeme s odděleným odporem a kapacitou, tak i rozprostřené parametry. Tyto prvky zajišťují velmi přesné výsledky v případě hmotných konstrukcí. V modelech lze snadno začlenit okení konstrukce i automaticky a ručně řízené zdroje. Uvedeny jsou klíčové vzorce využívající Laplaceovu a Fourierovu transformaci. V minulých letech bylo vyvinuto simulační prostředí umožňující pracovat s libovolným modelem a zátěžemi s periodickou změnou. Výsledky simulací byly ověřeny měřením na různých budovách.

Klíčovou součástí teorie tolerancí je citlivostní analýza. Pro popisovanou problematiku to je citlivostní analýza přenosové funkce. V práci jsou uvedeny vzorce pro výpočet relativních diferenciálních citlivostí jednotlivých prvků modelu. Postup výpočtu je demonstrován na příkladu citlivostní analýzy modelu popisující změny teploty vzduchu v interiéru v závislosti na změnách venkovního vzduchu. Nejcitlivějším prvkem je okno a přestup tepla z interiéru do okolních konstrukcí. Aplikací tolerancí v dalším uvedeném příkladu je ukázána schopnost metody rychle nalézt parametry modelu a následně parametry stavební konstrukce pro požadované změny teploty vnitřního vzduchu v letním období.

Popsaná metoda dokazuje potenciál metod vycházejících z teorie tolerancí. Na závěr jsou uvedeny i omezení vyplývající z použití diferenciálních citlivostí.

Klíčová slova: počítačová simulace, dynamické chování, tepelná síť, teorie tolerancí, citlivostní analýza

Keywords: computer simulation, dynamic behavior, thermal network, theory of tolerances, sensitivity analysis

CONTENTS

1. INTRODUCTION

Computer simulation deals with an experimentation on a computer based model of complex systems. The level of satisfaction with simulated outputs decides what model parameters will be implemented in the practice for solving investigated problem situation. There are two possibilities how to obtain outputs describing the system behavior – the simulation and experimentation. The simulation approach has the following advantages: *costs* – experiments on real systems are expensive or impossible; *time* – high information flow in short time period, see figure 1; *replication* – the simulation is repeatable; *safety* – the designed system can be tested in extreme conditions [1].

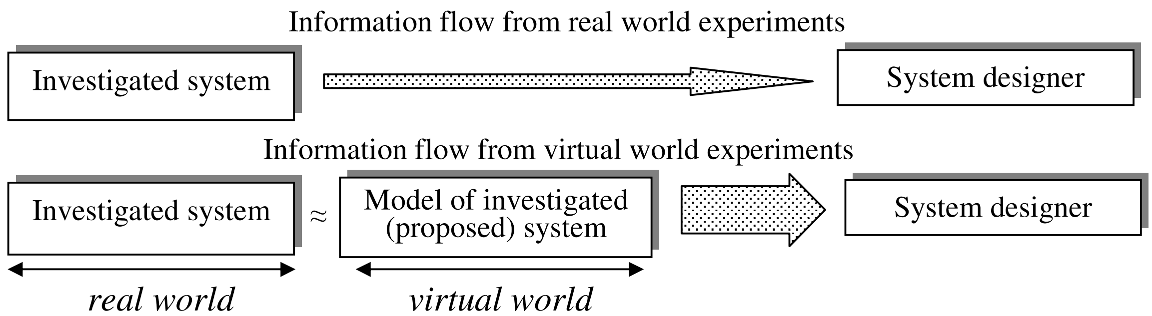


Figure 1. Information flow from simulation

The most important results from computer simulation are for project oriented companies where the main output of their work is the project. The project is an unique process and therefore it makes sense to perform simulation before realization of the designed system. The projects are characteristic production processes in a construction industry. The computer simulation in this industry became the important part of the design work.

The important example of using the computer simulation approach is the simulation of **thermal dynamic behavior of buildings and energy systems**. Simulation is performed during certain time period and outputs are time-dependent changes of internal air temperatures, surface temperatures or heat flows [2]. The simulated variables are the basis for the decision about the use of main building structure elements and HVAC systems in the buildings. This decision will substantially influence the energy consumption in designed or existent buildings and have the influence on the quality of the internal environment. Good environmental quality improves the productivity of workers [3] and has the important economic effect. Good designed building is the example of the balance between the energy conservation and the indoor quality as well as the balance between investment and running costs.

1.1 Methods of designs

It is possible to build the dynamic model of the building but also models of project oriented companies were built. Simulated variable in this case is the amount of the finished work in the project. The results from the simulation demonstrate the fact that the amount and the quality of the finished works depend on: the productivity of human resources, the productivity of other resources (e.g. software) and the knowledge. The result is: Our efforts should be focused on the development of methods for increasing the productivity of the design works. The possible strategy is to use the simulation methods but also to develop the simulation methods and programs for the fast calculation of the required parameters of the designed system. The description of methods as the processes is drawn in Figure 2.

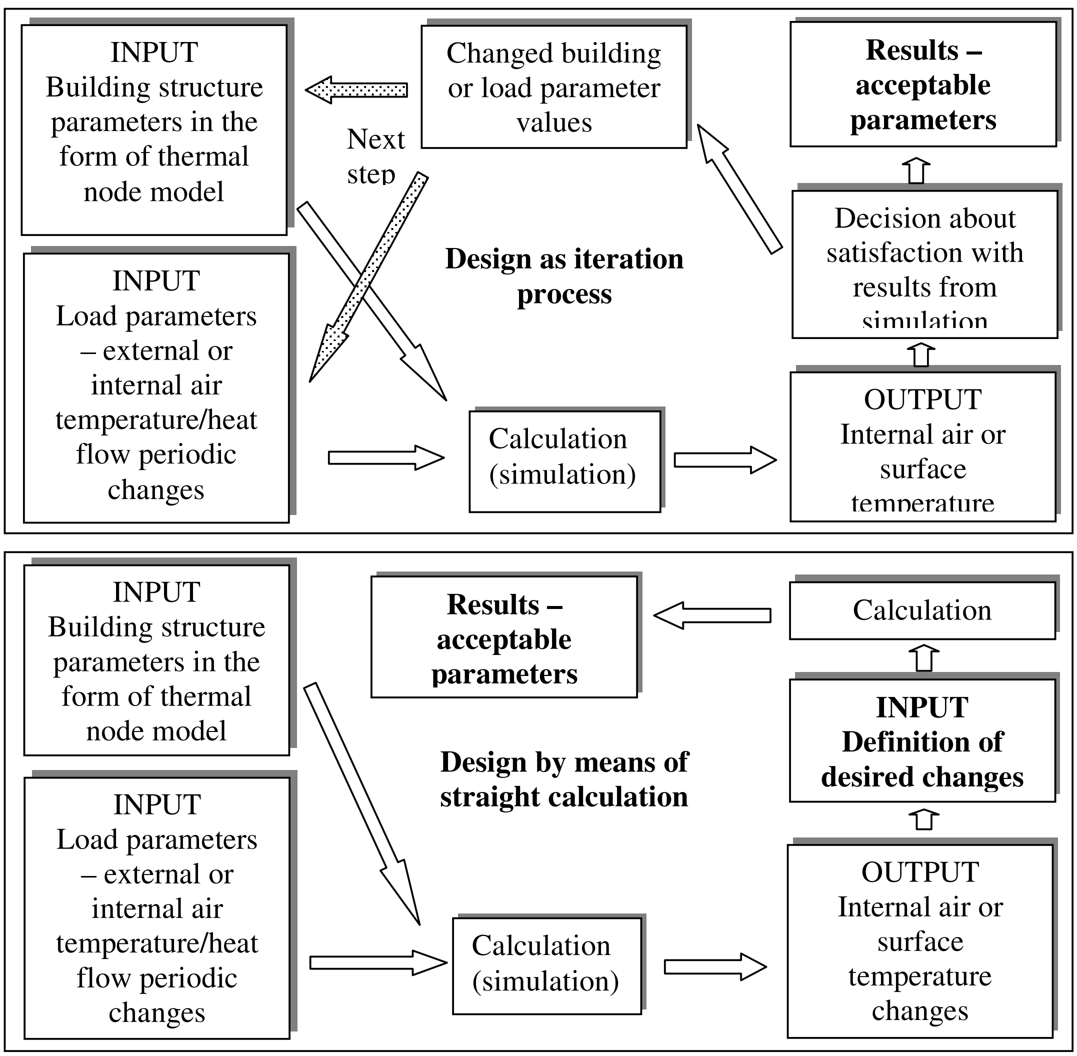


Figure 2. Explanation of the difference between a standard design approach and new design methods

2. DESCRIPTION OF METHODS

2.1 Thermal network technique

The response of the system on the change of input (disturbance) variable is

$$y(t) = f(t) * x(t) = \int_0^t f(t-\tau) \cdot x(t) \; d\tau \tag{1}$$

where

y(t) is response variable of the system,
f(t) is characteristics of the system,
x(t) is disturbance variable,
* is convolution.

For solving equation (1) it is possible to use Laplace transform and the image of variable *x(t)* is

$$x(p) = \int_0^\infty x(t) \cdot e^{-p \cdot t} \; dt \tag{2}$$

where

p is Laplace operator.

The response after the transform is

$$Y(p) = F(p) \cdot X(p) \tag{3}$$

The response in time domain is calculated from known images

$$y(t) = \mathcal{L}^{-1}\{F(p) \cdot X(p)\} \; . \tag{4}$$

Next part will deal with the theory for description of thermal dynamic behavior of buildings and energy systems.

Characteristics *F(p)* is derived from the next equations, see [4]

$$[Q] - [h] \cdot [T] = 0 \tag{5}$$

where

[Q] is matrix of flows,
[T] is matrix of temperatures,
[h] is system matrix describing the structure of system.

Matrix of temperatures and flows is

$$\begin{bmatrix} T_1 \\ \\ \\ T_n \end{bmatrix} = \frac{1}{\Delta} \cdot \begin{bmatrix} \Delta_{11} & & \Delta_{1m} \\ & & \\ & & \\ \Delta_{n1} & & \Delta_{nm} \end{bmatrix} \cdot \begin{bmatrix} Q_1 \\ \\ \\ Q_n \end{bmatrix} \tag{6}$$

In case of $Q_2 = Q_n = 0, \quad Q_1 \neq 0$
the temperatures in output and input nodes are

$$T_k = \frac{1}{\Delta} \cdot \Delta_{1k} \cdot Q_1 \tag{7}$$

$$T_1 = \frac{1}{\Delta} \cdot \Delta_{11} \cdot Q_1 \tag{8}$$

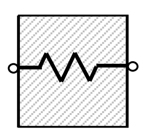
and the transfer function is

$$F(p) = \frac{T_k}{T_1} = \frac{\Delta_{1k}}{\Delta_{11}}. \tag{9}$$

More detailed description of the theory and the solution of equations in frequency domain by means of Fourier transform is described in [4, 5].

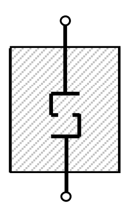
The network is built as an electric circuit and includes these elements:

thermal resistance



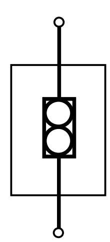
$$Q = \frac{1}{R_t} \cdot \Delta T \tag{10}$$

thermal capacitance



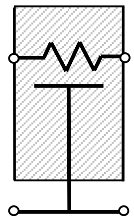
$$Q = C_t \cdot \frac{dT}{dt} \tag{11}$$

source of heat flow



$$Q = G \cdot T \tag{12}$$

distributed parameter – the element includes the properties of thermal capacitance and thermal resistance; increases the accuracy of simulation [6]; suitable for heavyweight walls.



In case of distributed parameters the equation (11) is added to the system matrix

$$\left[h_{ij}\right] = \frac{1}{\rho_0 \cdot \sinh \tau l} \cdot \begin{bmatrix} \cosh \tau l & -1 \\ -1 & \cosh \tau l \end{bmatrix} \tag{13}$$

where

$$\rho_0^2 = \frac{R_t}{p \cdot C_t} \quad \text{and} \quad \tau^2 = p \cdot R_t \cdot C_t \tag{14}$$

where

R_t and C_t are specific thermal resistance and specific thermal capacitance,
i,j are nodes where distributed parameter element is connected to the model.

The description of the system with main subsystems is in Figure 3. These subsystems are included in the network models according to the solved problem situation.

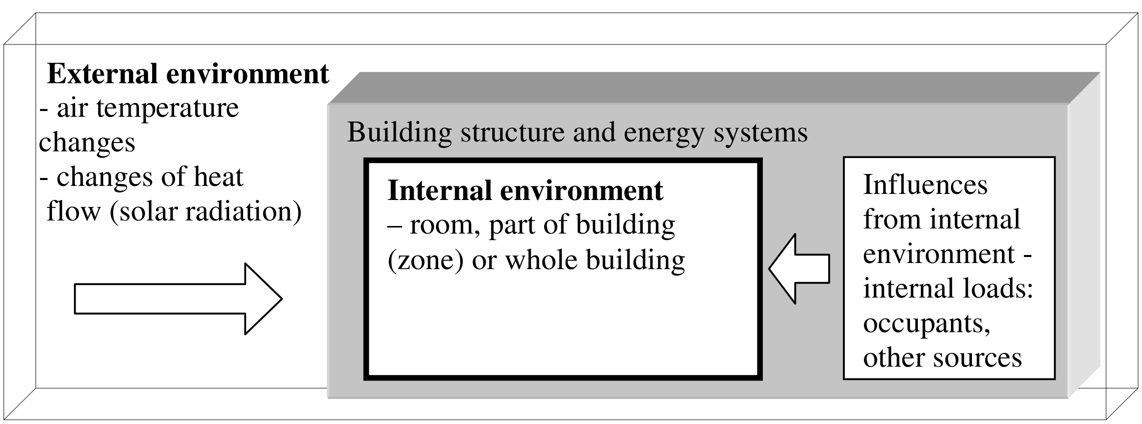


Figure 3. System external environment – building structure – energy system – internal environment

During last years many models have been developed for solving different problem situations [7-9]. The models use both types of transforms - Laplace transform for the step changes (e.g. switch on/off of the heating system), Fourier transform for the slow periodic changes (external environment changes - air temperature, solar radiation; internal environment changes – heating, internal loads from equipments and occupants). The example of the model describing the internal air temperature changes according to changes of external air temperature is in Figure 4.

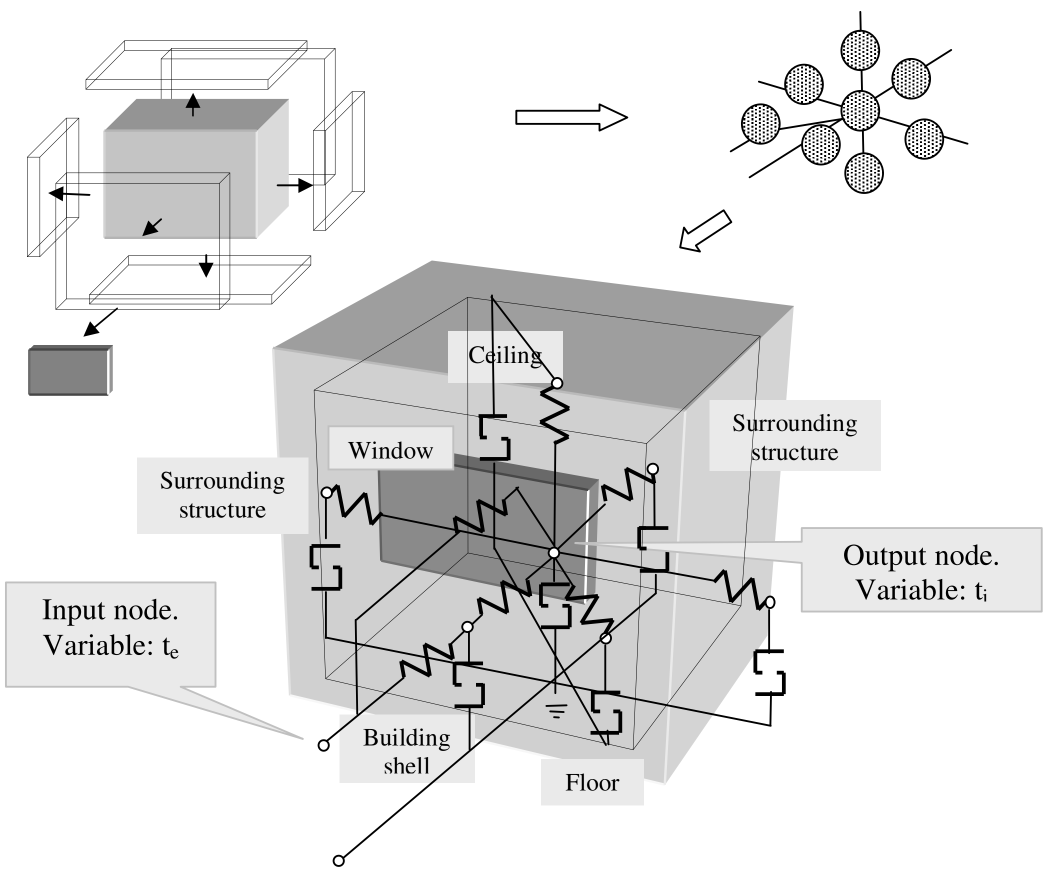


Figure 4. Building structure and full model with separated elements for all parts of building structure. Input: external air temperature changes, output: internal air temperature changes.

2.2 Sensitivity analysis of networks

Introducing the sensitivity analysis and the theory of tolerances to the simulation program brings the fast calculation of the desired changes of the output parameters and solve the problems described in the introduction [10].

The relative sensitivity is expressed in equation (15), [11]

$$Sr_x^F(p,x) = \frac{\partial \ln F(p,x)}{\partial \ln x} = \frac{\partial F(p,x)}{\partial x} \cdot \frac{x_0}{F_0} \tag{15}$$

where

x is model parameter,

x_0 and F_0 are nominal values.

The system function after assessing algebraic complements is

$$F(p,x) = \frac{N(p,x)}{D(p,x)} \tag{16}$$

where

N(p,x) and *D(p,x)* are polynomials with variable p and x.

The relative sensitivities can be calculated from equation (17), [12]

$$Sr_x^F(p,x_i) = x_i \cdot \left(\frac{N'}{N} - \frac{D'}{D} \right). \tag{17}$$

The expression (17) is the sensitivity function. The resultant value is a complex figure because $p = j.\omega$, where j is the imaginary unit and ω is radian frequency. Real part expresses the amplitude sensitivity and the imaginary part expresses the sensitivity of the phase delay to the change of the parameter x, see equation (18)

$$Sr_{x_i}^F(j\omega) = \operatorname{Re} Sr_{x_i}^F + j \operatorname{Im} Sr_{x_i}^F = \frac{\partial |F(j\omega, x_i)|}{\partial x_i} \cdot \frac{x_i}{|F(j\omega, x_i)|} + j \frac{\partial \{\arg F(j\omega, x_i)\}}{\partial x_i}. \tag{18}$$

The importance of both parts of the sensitivities is explained in Figure 5.

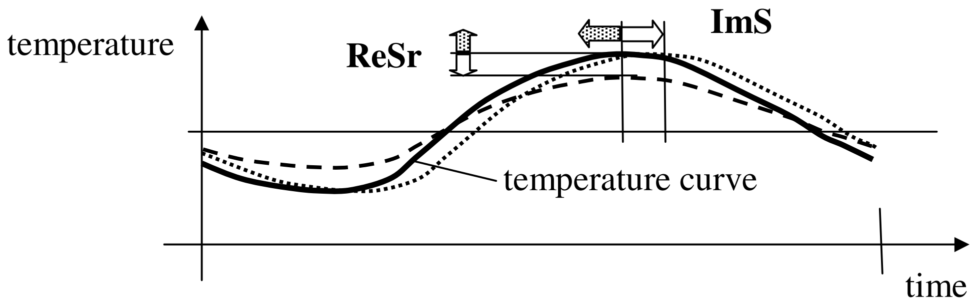


Figure 5. Explanation of real and imaginary part of sensitivities

2.3 Application of theory of tolerances

The tolerance of the system function *F* is a differential defined by the equation (19), [11]

$$\Delta F = \sum_{i=1}^{N} S_{x_i}^{F} \cdot \Delta x_i \; . \tag{19}$$

The relative tolerance makes use of relative sensitivities

$$\frac{\Delta F}{F} = \sum_{i=1}^{N} Sr_{x_i}^{F} \cdot \frac{\Delta x_i}{x_i} \; . \tag{20}$$

Equations (17) and (20) are theoretical basis for the fast design of new model parameters. The calculation of the relative sensitivities makes possible the program developed by the author for this purpose [13].

3. EXAMPLE

3.1. Calculation of relative sensitivity

The first example shows the calculations of relative sensitivities of model elements. The model describes the changes of internal air temperature according to changes of external air temperature. The building structure and adequate thermal model is in Figure 6. Resultant values of relative sensitivities are depicted in Figure 7. The mot sensitive elements are the thermal conductivity of the window and the element that represents the conductivity from the internal node to the surrounding structures. The sensitivity values were calculated for certain range of the nominal values and are drawn in Figure 8. The values are different from every input value. This result supports the expected fact that sensitivity is not constant.

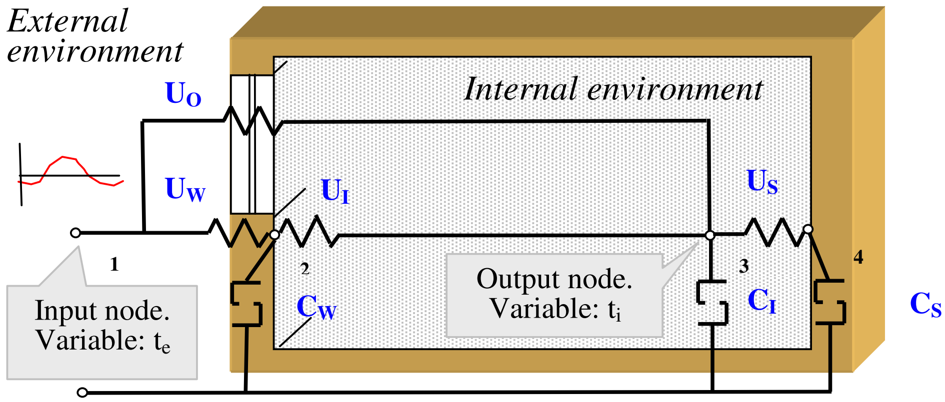


Figure 6. The part of building structure and the thermal model

Table 1. Input values of model parameters

U_W	U_O	U_I	U_S	C_W	C_I	C_S
	[W . K^{-1}]				[Wh.K^{-1}]	
5.3	14	128	739	611	395	5998

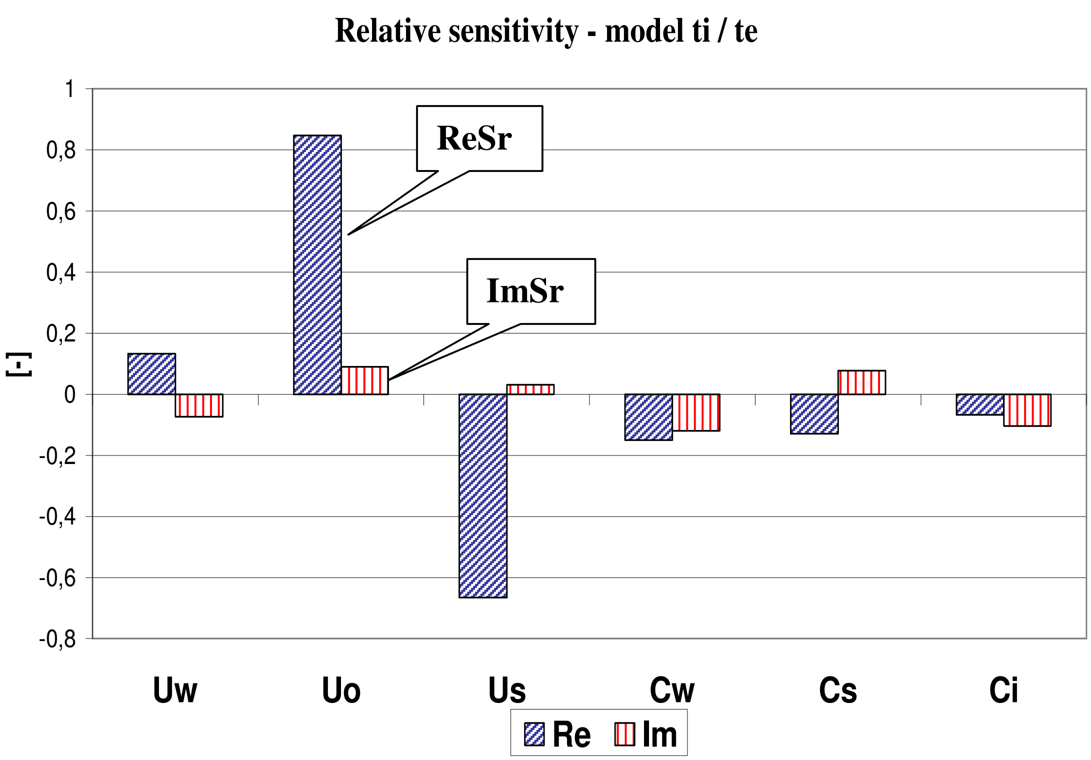


Figure 7. Real and imaginary part of relative sensitivities

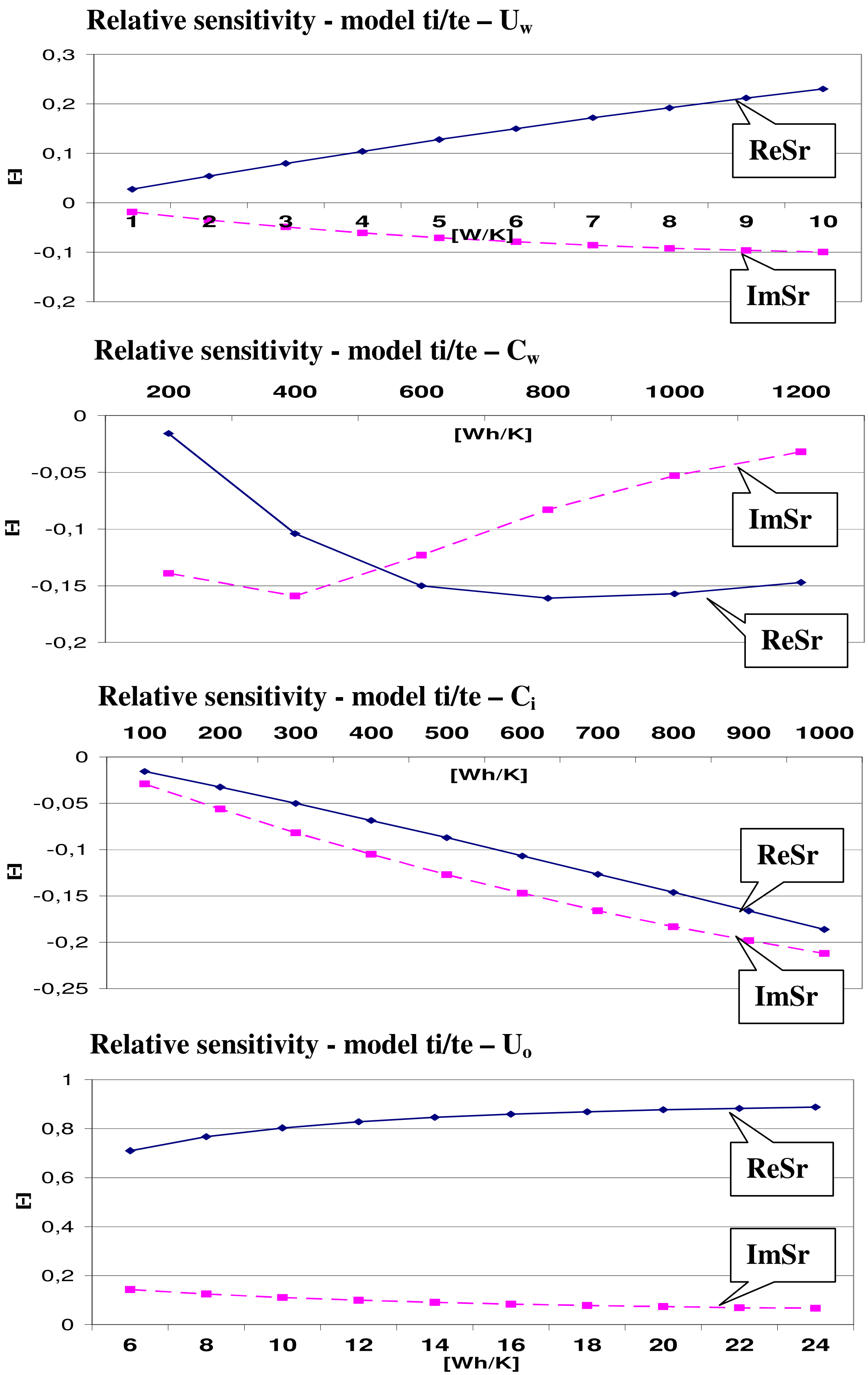


Figure 8. Relative sensitivities for different parameter values

3.2 Application of theory of tolerances

This example will present the fast calculation of building structure parameters in the office in summer period. For the analysis two models were used, the first one is the same model as in above mentioned example, the second one is depicted in Figure 9. The five-node model allows us to assess internal air temperature changes evoked by external changes of the heat flow, in this case it means by the solar radiation. The elements G_o and G_w distribute the solar radiation to the building shell and the window. G_L represents the heat flow that is caused by the convection [14].

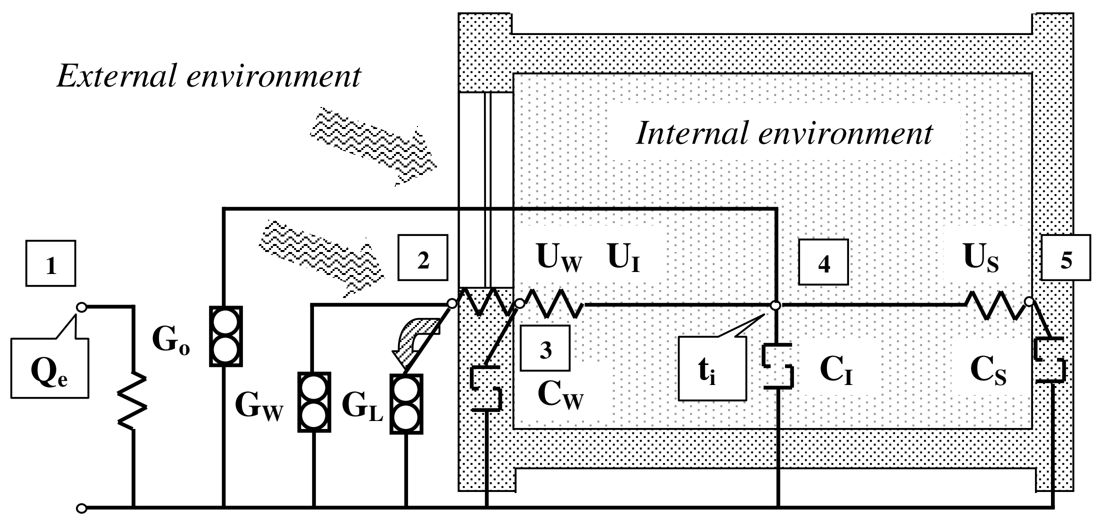


Figure 9. Model for the calculation of internal air temperature in summer

The curves of both load variables during the summer day for south orientation are drawn in Figure 10.

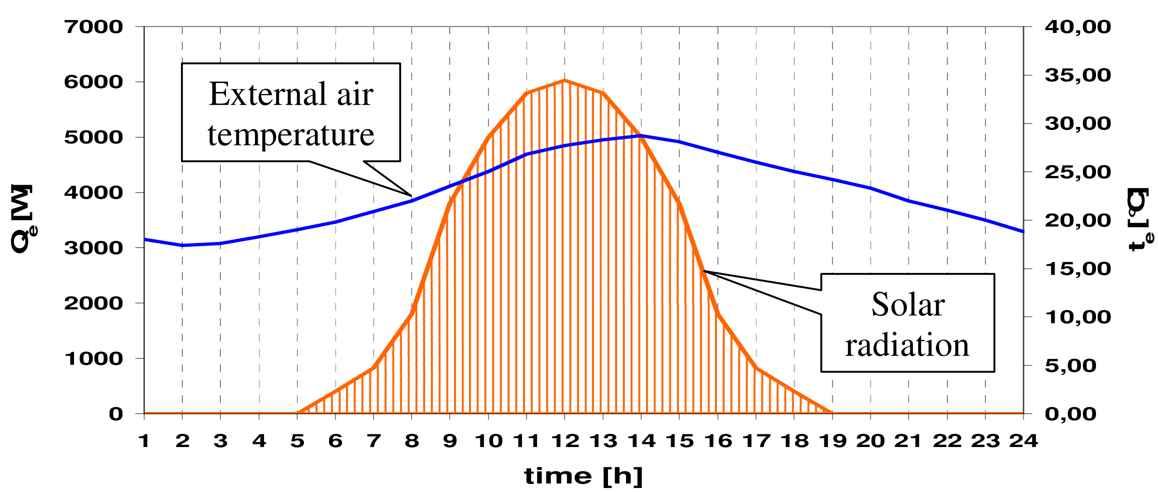


Figure 10. The time-dependent load variables for south oriented façade

The resultant values of predicted internal air temperature for initial model values are depicted in Figure 11. The calculated temperature peak is 27,3°C. The acceptable temperature in the analysed office is 26°C. *The goal of this study is to find the parameters of the passive elements of the system that will ensure the desired temperature 26°C.* For this purpose the real parts of the relative sensitivities were calculated, see figure 12. Only real parts were considered because we are interested only in the amplitude change. Also only parameters that can be changed during the design process were chosen. The most sensitive model parameter is the heat source element that describes the heat flow thorough the window. Another important element is also the thermal capacitance of the internal building structures.

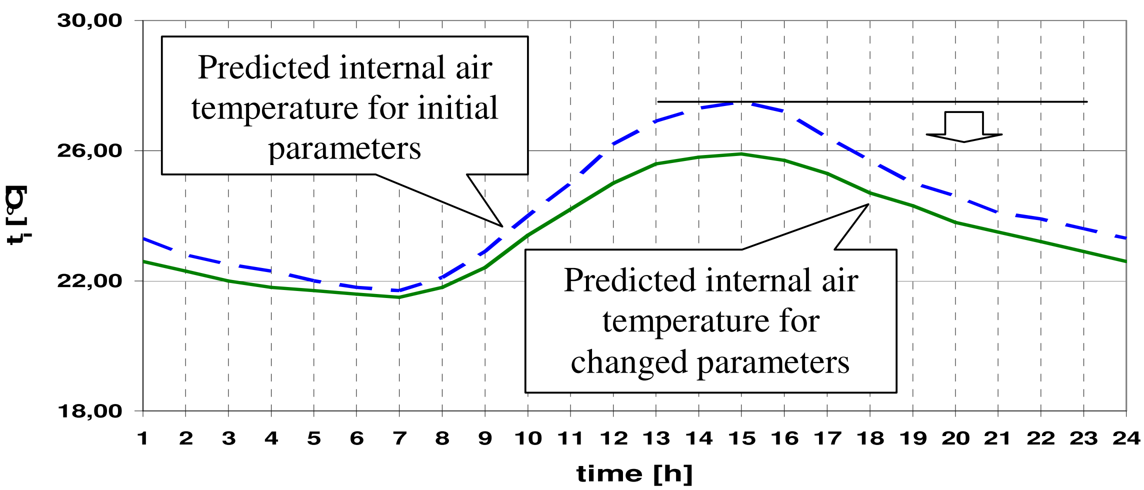


Figure 11. Temperature curve for initial parameter values and for calculated values

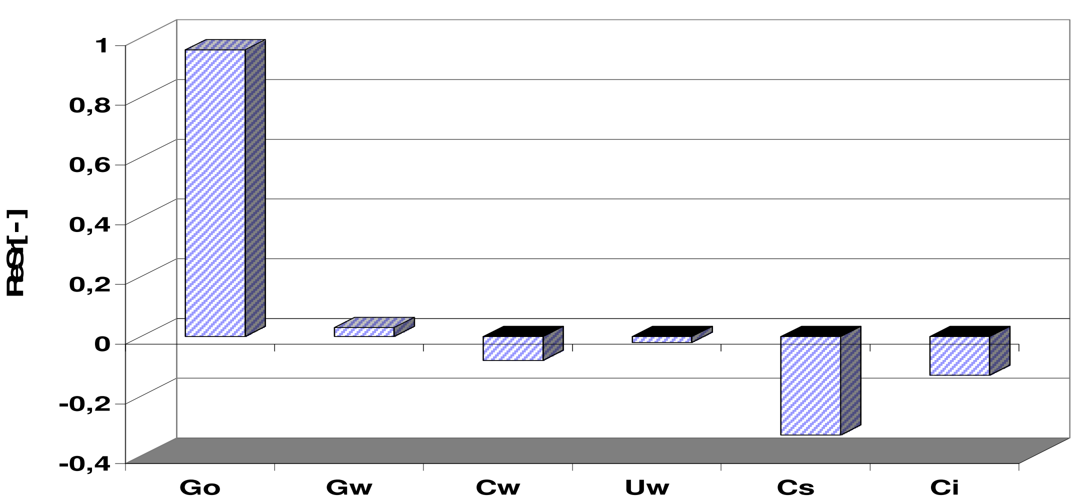


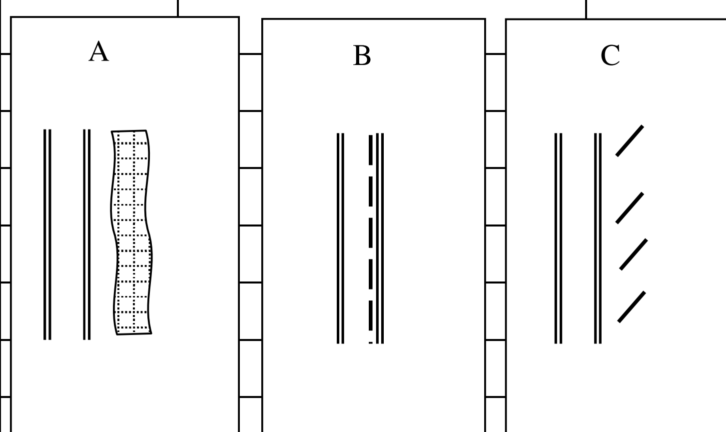
Figure 12. Resultant values of real part of relative sensitivities

The elements G_o was chosen and its new value was computed from equation (20) and the relative sensitivity value. Initial input values and calculated values are in Table 2. For the verification of the result, air temperature curve were calculated with new value of the parameter G_o. Maximum air temperature in the room achieved 26°C.

The next step of the process is to propose the physical parameters of the building structure. Three variants were considered. All arrangements ensure desired goal – the decrease of the air temperature peak under 26°C. The description of variants is in Table 2. In this case study, the change only G_o value was considered. Other solutions can be the combinations of the change of G_o value and other parameter values, e.g. G_W and $G_{S.}$ The final decision has to consider also architectural and economic limits of the chosen solutions.

Table 2 Input and new calculated model values

	Input parameter values		Calculated parameter values	Strategy - parameter value can be reached by using:	New parameter values
G_O	- 0.292	[W/K]	- 0.219	A. drapes – shading coefficient s = 0.583 [-]	- 0.209
				B. coating on glass surface (Reflex glass) s = 0.62	- 0.222
				C. venetian blind (internal) s = 0.551	- 0.198
G_W	- 0.333	[W/K]			
G_L	96	[W/K]			
U_W	2.6	[W/K]			
U_I	51.2	[W/K]			
U_S	485.3	[W/K]			
C_W	93.5	[Wh/K]			
C_I	308	[Wh/K]			
C_S	1239	[Wh/K]			



4. CONCLUSIONS

The described method can be used for the fast design of the element parameters and consequently the structure parameters. A computer program for computing relative sensitivities has been developed. The important fact is the application of differential sensitivities. It means that we can work only with relatively small changes of the parameter values. It depends on the sensitivity characteristics. For G_0 sensitivity values it is possible to find stable results of the relative sensitivities for the wide range of the nominal values. In the described case study, the investigated parameter was changed by 25,5 %. The comparison of the calculated air temperature peak for the new parameter value and the desired air temperature peak shows good accuracy of the method.

The application of the tolerance theory to the simulation methods improves the building and HVAC design process and ensures the quality of the simulation outputs. The implementation of these methods to the simulation programs also improves the productivity in the design companies. Main features of good designed building are:

- low energy consumption,
- high quality of internal environment.

Presented method has the impact on both points.

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